ELEMENTARY PARTICLES
and
WEAK INTERACTIONS
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Foreword

The substance of these notes comes from published and unpublished research of the undersigned. In form and presentation these notes follow largely a series of six lectures given by one of us (T.D. Lee) at Brookhaven National Laboratory during January 1957. The lecture notes were originally edited by Drs. L.C.L. Yuan, B.H. McCormick, W. Chinowsky, and R.K. Adair, to all of whom grateful acknowledgment is hereby made. Dr. Yuan is especially to be thanked for the time and advice he generously contributed in the process of the later expansion and changes in the notes.

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I. GENERAL REVIEW

1. Introduction

In these discussions we shall try to review some general patterns of the interactions between various elementary particles and to study some general questions concerning the symmetry properties of these particles. The first natural question that one would like to ask is, what precisely constitutes an elementary particle? Suppose a new particle is observed, how do we know that it is an elementary particle and not merely a composite system consisting of some already known elementary particles? The answer is that we do not know. Nevertheless, the term “elementary particle” is well defined in a negative sense. We believe we understand what is meant by an atom, a molecule, and a nucleus. Any small particle that is not an atom, not a molecule, and not a nucleus (except the hydrogen nucleus) is called an elementary particle.

At first sight these so-called elementary particles form a very inhomogeneous group. The mass of a particle may be large (as $M \approx 1321$ Mev) or may be very small (as the mass of $\gamma = 0$). The lifetime may be very short (as the lifetime of $\Sigma^0 \approx 10^{-20}$ sec) or may be very long (as in the case of a proton which has infinite lifetime). The particle may be electrically charged or may be neutral. Except for a very few of them, we do not know the spin and parity of these particles. Again except for those between a very few of these particles, we do not know their interactions. In fact there is still confusion as to the identity of the particles that have already been observed. Some of these particles that look very different may turn out to be the same particles. Some of them that look very similar may indeed be several different particles. In spite of this almost complete lack of understanding, some general patterns and general rules have been found. These properties will be our main topics of discussion.

2. Review of the Classifications of Interactions

We find that the interactions between these elementary particles fall into four distinct groups:

(i) Strong interactions. This group includes the forces responsible for the production and the scattering of nucleons, pions, hyperons, and K-mesons. It is characterized by a coupling constant of the order of magnitude $1 (f^2/\hbar c \approx 1)$.

(ii) Electromagnetic interactions. These are characterized by the coupling constant, $e^2/\hbar c = 1/137$.

(iii) Weak decay interactions. These are characterized by a dimensionless coupling constant, $G^2/\hbar c = 10^{-14}$.

(iv) Gravitational interactions. The gravitational interaction can be characterized by a dimensionless coupling constant $Gm^2/\hbar c = 2 \times 10^{-39}$ with $G$ as Newton's
gravitational constant and \( m \) chosen to be the mass of the proton. The gravitational interaction will not be further discussed.]

Among these three interactions only the electromagnetic interaction is well understood. About the other two groups, the strong interactions and the weak decay interactions, we know really very little. Nevertheless, we believe they possess certain symmetry properties.

3. Invariance Properties and Conservation Laws that are USUALLY Accepted as Exact

We shall now list those symmetry properties and conservation laws that were, before the end of 1956, generally accepted to be valid for all three groups of interactions. These are:

(i) Conservation of energy and momentum. This follows from the invariance under translations in the four-dimensional space. The infinitesimal translations in space are represented by the energy-momentum operators \( P_\mu \). Thus, the homogeneity of space implies the conservation of energy and momentum.

(ii) Invariance under the proper Lorentz transformation. A proper Lorentz transformation is a Lorentz transformation without either space inversion or time reversal. This invariance implies, among other things, the conservation of angular momentum.

(iii) (\( ? \)) Invariance under space inversion \( P \) (or, the conservation of parity). \( P \) is a transformation which changes \( r \to -r \); \( t \to +t \); and particles \( \to \) particles.

(iv) (\( ? \)) Invariance under time reversal \( T \) (i.e., \( r \to +r \); \( t \to -t \); and particle \( \to \) particle).

(v) (\( ? \)) Invariance under charge conjugation \( C \). The charge conjugation operator \( C \) changes a particle to its antiparticle, but leaves \( r \to +r \) and \( t \to +t \).

(vi) Conservation of charge \( Q \). This conservation law is related to the invariance under gauge transformation.

(vii) Conservation of heavy particle number \( N \).

Because of our belief that these invariance principles and conservation laws are valid for all interactions, strong, electromagnetic, and weak interactions, the most important characteristics of an elementary particle are its mass, spin, parity, charge, and heavy particle number. The question of its detailed dynamical behavior, such as scattering cross sections, production cross sections, decay modes, and lifetimes, is usually studied under specific assumptions about these intrinsic characteristics of the particle.

However, recent experiments\(^1\) have led to a different picture of these symmetry principles. In particular, they show that the invariance under space inversion and charge conjugation are not valid in certain weak interactions. We shall return to these questions in detail in our later discussions.


4. Invariance Properties That Are Approximately True

It has been found that in addition to the above symmetry principles there are some further approximate invariance principles. These are the conservation of isotopic spin \( I \) and the conservation of \( I_z \) (or strangeness \( S \)).\(^3\)\(^4\) We shall discuss the properties of these approximate conservation laws.

**A. PION-NUCLEON SYSTEM**

We review briefly the concept and the validity of isotopic spin in a system consisting of pions and nucleons. We describe the proton and the neutron as a spinor in the isotopic spin space:

\[
\begin{array}{ccc}
    & I & I_z \\
    p & \frac{1}{2} & \frac{1}{2} \\
n & \frac{1}{2} & -\frac{1}{2} \\
\end{array}
\] (1.1)

The pions are considered as forming a vector in the isotopic spin space according to the assignments:

\[
\begin{array}{ccc}
    & I & I_z \\
\pi^+ & 1 & 1 \\
\pi^0 & 1 & 0 \\
\pi^- & 1 & -1 \\
\end{array}
\] (1.2)

The total isotopic spin of a pion-nucleon system is given by the quantum mechanical sum of the isotopic spin vectors of all particles. If the total isotopic spin \( I \) is conserved, then all physical observables are left invariant under a rotation in the isotopic spin space. We find that for the strong interactions this is indeed true.

**Role of the Electromagnetic Field**

From the assignments (1.1) and (1.2) we find the following empirical relation between the charge \( Q \) and \( I_z \) for a pion-nucleon system:

\[
Q = I_z + (N/2)
\] (1.3)

where \( N \) is the heavy particle number \((N=1 \text{ for } p \text{ and } n; \ N=0 \text{ for } \pi)\). Because of this relationship the electromagnetic field destroys the invariance under arbitrary isotopic rotations except along the \( z \) axis. Thus the total isotopic spin quantum number \( I \) is not conserved in the electromagnetic interaction, while \( I_z \) is still conserved. To see this in detail let us represent the field of the nucleon by a two-component spinor function

\[
\psi = \begin{pmatrix} \psi_p \\ \psi_n \end{pmatrix}.
\]


The corresponding isotopic spin operator for a nucleon is then given by

\[ I_{\text{nucleon}} = (\frac{1}{2}) \int \psi^\dagger \gamma_4 r \psi d^3 r \]  

(1.4)

with \( \tau \) representing the \( 2 \times 2 \) Pauli matrices. The electromagnetic field is described by the Hamiltonian

\[ H_\gamma = - \int j_\mu A_\mu d^3 r \]  

(1.5)

where \( A_\mu \) is the electromagnetic potential and \( j_\mu \) is the electric current given by

\[ j_\mu = (\frac{1}{2}) i e [\psi \gamma_4 \gamma_\mu (1 + \tau_3) \psi] . \]  

(1.6)

We therefore immediately find that \( I_z \) commutes with \( H_\gamma \),

\[ [I_z, H_\gamma] = 0 , \]  

(1.7)

which means that \( I_z \) is conserved in the electromagnetic interactions. On the other hand, \( I_x, I_y, \) and \( F \) (= \( I_x^2 + I_y^2 + I_z^2 \)) all do not commute with \( H_\gamma \). In fact, under a rotation in the isotopic spin space the noncommuting part in \( H_\gamma \) behaves like the third component of an isotopic spin vector. Consequently, we find that the electromagnetic interactions can violate conservation of \( I \) by

\[ \Delta I = 0, \pm 1 , \]  

(1.8)

whereas \( I_z \) is still conserved, i.e.,

\[ \Delta I_z = 0 . \]  

(1.9)

Similarly, we can apply the above arguments to any pion-nucleon system and obtain selection rules identical to Equations (1.8) and (1.9). The breakdown of the total isotopic spin by the electromagnetic fields accounts, at least in a qualitative sense, for the mass difference between the neutron and the proton (\( \Delta m \approx 2.5 m_e \)) and the mass difference between \( \pi^0 \) and \( \pi^0 \) (\( \Delta m \approx 9 m_e \)).

B. CONSERVATION OF I SPIN IN SYSTEMS INVOLVING OTHER ELEMENTARY PARTICLES

It is natural (in fact, it is almost necessary) to assume that \( I \) spin is conserved for all strong interactions. Let us, as an example, consider the following strong interaction:

\[ \pi^- + p \rightarrow \Lambda^0 + K^0 . \]  

(1.10)

Suppose, for the sake of argument, that this reaction does not conserve the isotopic spin vector \( I \). By going through the virtual processes of emission and absorption of \( \Lambda^0 + K^0 \), we would find that isotopic spin is not a good quantum number for the \( \pi^- + p \) system. Since the reaction (1.10) is a strong interaction, the violation of isotopic spin conservation in the pion-nucleon system will be strong. From our previous discussions we know that this is not the case. Consequently, reaction (1.10) is expected to conserve the isotopic spin. Similarly, we expect that the conservation law of isotopic spin should be valid for all strong interactions.

Accepting, then, that isotopic spin is conserved in all strong interactions, what must be the isotopic spin assignment of each of the other elementary particles in the
various strong reactions, and what will be the relationship between $I_\epsilon$ and $Q$ for these new particles? Consider a particle $A$ which is involved in the strong interactions. As the strong interactions conserve isotopic spin, we must assign to the particle $A$ an isotopic spin quantum number $I$ and its $z$ component $I_z$. By a rotation in the $I$ spin space, we generate $2I + 1$ states that are degenerate with respect to the strong interactions. We shall show that these isotopic spin multiplets are expected to be states of charges differing from each other by unity.

Let us consider the charge $Q$ of one of these states. It is easy to see that if $Q$ is a function of $I_\epsilon$ then it must be a linear function of $I_z$. This follows from the fact that in a system of these particles $A$, the total charge $Q$ and the total $I_z$ of the system are given, respectively, by the linear sum of the $Q$ and $I_z$ of the individual particles. Consequently $Q$ can at most be a linear function of $I_z$, i.e.,

$$Q = \alpha_A I_\epsilon + (\gamma_A/2) \quad (1.11)$$

where $\alpha_A$ and $\gamma_A$ are quantities independent of $I_z$.

We want to demonstrate that the constant $\alpha_A$ must be 1. Let us consider a strong interaction of the form

$$\text{pions + nucleons} \rightarrow A + B . \quad (1.12)$$

From Equation (1.3) and the assumption that $I_\epsilon$ is conserved in reaction (1.12), we have

$$(Q)_\text{total} = (I_\epsilon)_\text{total} + [(N)_\text{total}/2] = Q_A + Q_B . \quad (1.13)$$

For simplicity, suppose that

$$I_A = I_B = \frac{1}{2} ,$$

and consider first the state where the $z$ components of both $I$ are up $\uparrow A \uparrow B$. We have then $(I_z)_{\text{total}} = 1$. The total charge $Q$ is

$$Q = Q(A) + Q(B) \quad (1.14)$$

where $Q(A)$ and $Q(B)$ are the charges corresponding to the states $\uparrow A$ and $\uparrow B$. Since the total $I$ spin is conserved, we must have also the corresponding state with $(I_z)_{\text{total}} = 0$, which is $(1/\sqrt{2})(\uparrow A \downarrow B + \downarrow A \uparrow B)$. Furthermore, by considering the left-hand side of the reaction (1.12) and by using Equation (1.13) we conclude that the corresponding total charge for this state, $(I_z)_{\text{total}} = 0$, must be $Q - 1$ with $Q$ given by Equation (1.14). Hence we have

$$Q(A) + Q(B) = Q(B) + Q(A) = Q - 1 .$$

By comparing with (1.11), we have

$$\alpha_A = \alpha_B = 1 .$$

In a similar way, this result $\alpha_A = \alpha_B = 1$ can be generalized to particles of $I$ values other than $\frac{1}{2}$. It therefore follows that for all the strange particles\textsuperscript{3} which interact strongly with the pion-nucleon system, we must have the relation,

\textsuperscript{3}The term "strange particle" is again defined in a negative sense. It applies to any elementary particle which participates in the strong interactions but is not a nucleon or a pion.
\[ Q = I_z + (Y/2) \]  
(1.15)

where \( Y \) is a quantity independent of \( I_z \).

From this relation, Equation (1.15), there follow a number of consequences which we list below:

(i) In all these discussions we shall assume that there does not exist a doubly charged particle. Consequently from (1.15), \( I \) is limited to the values

\[ I = 0, \frac{1}{2}, 1 \]  
(1.16)

for a single particle.

(ii) If we assume as consistent with experimental evidence that there are no charged particles of about the same mass as the \( \Lambda^0 \), then we must have

\[ I_{\Lambda^0} = 0. \]  
(1.17)

(iii) Consider again the reaction

\[ \pi^- + p \rightarrow \Lambda^0 + K^0. \]  
(1.10)

The total isotopic spin of the left-hand side is \( (I)_{\text{total}} = \frac{1}{2} \) or \( \frac{3}{2} \). By using the above properties (i) and (ii) we conclude

\[ I_K = \frac{1}{2}, \]  
(1.18)

and for the \( I_z \) component

\[ (I_z)_K = -\frac{1}{2}. \]  
(1.19)

From Equation (1.15) the corresponding particle with \( I_z = +\frac{1}{2} \) must be a positively charged particle \( K^+ \). By applying the charge conjugation operator \( \mathcal{C} \) to \( K^+, K^0 \), we generate two other \( K \)-particles, \( \bar{K}^- \) and \( \bar{K}^0 \), with

\[ I_{\bar{K}} = \frac{1}{2}, \quad (I_z)_{\bar{K}^0} = \frac{1}{2}, \quad \text{and} \quad (I_z)_{\bar{K}^+} = -\frac{1}{2}. \]  
(1.20)

Thus we uniquely determine the \( I \) spin of the \( K \)-particles and deduce that there must be at least four such particles.

(iv) In a manner entirely analogous to the above, we can use the reaction, say,

\[ \pi^- + p \rightarrow \Sigma^- + K^+. \]  
(1.21)

By comparing the \( I_z \) value on both sides we conclude that

\[ (I_z)_{\Sigma^-} = \frac{1}{2}. \]

Assuming that there is no doubly charged particle we have

\[ (I_z)_{\Sigma^0} = 1, \]  
(1.22)

which implies a triplet \( \Sigma^+ \) and \( \Sigma^0 \).

**Role of Electromagnetic Interactions**

Now since we have determined the relationship between charge and \( I_z \), Equation (1.15), for all the strange particles, we can apply the same argument concerning the role of electromagnetic interaction as that used in the pion-neucleon system. In
an entirely similar way it can be shown that the electromagnetic interaction conserves $I_z$, but not the total isotopic spin $I$. The selection rule is

$$\Delta I = 0, \pm 1$$  \hspace{1cm} (1.23)

and

$$\Delta I_z = 0.$$  \hspace{1cm} (1.24)

Consequently, for example, $\Sigma^0$ is unstable against $\gamma$ emission:

$$\Sigma^0 \rightarrow \Lambda^0 + \gamma.$$  \hspace{1cm} (1.25)

Since $I_z$ is conserved in the electromagnetic interaction as well as in the strong interactions and since the charge $Q$ and the heavy particle number $N$ are known to be conserved in all of these interactions, any linear combinations of these three quantities will be conserved in both the strong and the electromagnetic interactions. In particular, it is useful to define

$$S/2 = Q - I_z - (N/2), \text{ and } \tau = S + N.$$  \hspace{1cm} (1.26)

Because of the conservation of $I_z$, it follows that both $S$ and $\tau$ are conserved in the strong as well as the electromagnetic interactions. The quantity $S$ is called the strangeness quantum number.\textsuperscript{3,4} From the assignment of $I_z$ for various particles we find

$$(S)_p = (S)_p = (S)_n = 0,$$

$$(S)_{K^+} = (S)_{K^+} = +1,$$

$$(S)_{K^-} = (S)_{K^-} = -1,$$

$$(S)_{\Lambda^0} = (S)_{\Xi^+} = (S)_{\Xi^-} = -1.$$  \hspace{1cm} (1.27)

Both $S$ and $\tau$ do not vary with respect to different $I_z$ components in the same $I$ multiplets.

Let us consider now the assignment of $I$ to the cascade particle $\Xi^\pm$. The production of the $\Xi^\pm$ with two neutral $K$-mesons has been observed.\textsuperscript{4} However, we do not know whether these $K$-mesons are $K^0$'s or $\bar{K}^0$'s. Since $S$ is conserved in production processes, it follows that the $S$ of $\Xi^\pm$ is either zero or $\pm 2$ depending on the nature of these $K$-mesons. For $\Xi^+$, the charge $Q$ is $-1$ and the heavy particle number $N$ is $+1$. From Equation (1.26) we have

$$(I_z)_{\Xi^+} = -(3/2) - (1/2) (S).$$

In order that there be no doubly charged particle observed, we must have $|I_z| \leq 1$. Consequently, the isotopic spin assignment to $\Xi^\pm$ is

$$I_z = -1/2 \quad \text{and} \quad I_z = 1/2.$$  \hspace{1cm} (1.28)

Thus there should exist another $\Xi^0$ with $(I_z)_{\Xi^0} = +1/2$.

In Table 1 are listed various quantum numbers together with the masses and lifetimes for these particles.

<table>
<thead>
<tr>
<th>Particle</th>
<th>Lifetime (sec)</th>
<th>Mass (Mev)</th>
<th>Spin, Parity</th>
<th>I</th>
<th>I(_z)</th>
<th>S</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Xi^-)</td>
<td>((4.6 &lt; \tau &lt; 200) \times 10^{-10})</td>
<td>1321 ((\pm 3.5))</td>
<td>(\frac{1}{2}) (-\frac{1}{2})</td>
<td>-2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\Sigma^-)</td>
<td>(1.6 \times 10^{-10})</td>
<td>1196.65 ((\pm 0.35))</td>
<td>1</td>
<td>-1</td>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>(\Sigma^0)</td>
<td>(0.69 \times 10^{-10})</td>
<td>1189.7 ((\pm 0.25))</td>
<td>+1</td>
<td>-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\Sigma^+)</td>
<td>((&lt; 1) \times 10^{-11})</td>
<td>1188.65 ((\pm \frac{3}{2}))</td>
<td>0</td>
<td>-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\Lambda^0)</td>
<td>(3 \times 10^{-10})</td>
<td>1115</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>(\rho)</td>
<td>(10^3)</td>
<td>938</td>
<td>(\frac{1}{2})</td>
<td>(\frac{1}{2})</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>(\eta)</td>
<td>(10^3)</td>
<td>939</td>
<td>(\frac{1}{2})</td>
<td>(\frac{1}{2})</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>(K^-)</td>
<td>(1.2 \times 10^{-9})</td>
<td>494</td>
<td>(\frac{1}{2})</td>
<td>(\frac{1}{2})</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>(K^0)</td>
<td>(0.95 \times 10^{-10})</td>
<td>494</td>
<td>(1)</td>
<td>(1)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\bar{K}^0)</td>
<td>(0.95 \times 10^{-10})</td>
<td>494</td>
<td>(\frac{1}{2})</td>
<td>(\frac{1}{2})</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>(K^-)</td>
<td>(1.2 \times 10^{-9})</td>
<td>494</td>
<td>(\frac{1}{2})</td>
<td>(\frac{1}{2})</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>(\pi^\pm)</td>
<td>((&lt; 1) \times 10^{-11})</td>
<td>140</td>
<td>(0)</td>
<td>(\pm 1)</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>(\pi^0)</td>
<td>(10^{-13})</td>
<td>135</td>
<td>(0)</td>
<td>(0)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\mu^\pm)</td>
<td>(2.2 \times 10^{-4})</td>
<td>106</td>
<td>(\frac{1}{2})</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(e^\pm)</td>
<td>(5.1)</td>
<td>0.51</td>
<td>(\frac{1}{2})</td>
<td>(\frac{1}{2})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\nu, \bar{\nu})</td>
<td>(0)</td>
<td>0</td>
<td>(\frac{1}{2})</td>
<td>(\frac{1}{2})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\gamma)</td>
<td>(0)</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### C. WEAK INTERACTIONS

If we examine the weak interactions in detail, we find that they are divided distinctly into two groups:

(i) Those which are characterized by a nonconservation of \(I_z\) with

\[\Delta I_z = \pm \frac{1}{2}.\]  

(ii) Those which involve neutrinos, such as the \(\beta\) decay, the \(\mu\) decay, the \(\pi\) decay, the \(K_{\mu 3}\) etc.

These two groups seem to have completely different characteristics. Yet they share a remarkable common feature which is that the strengths of the coupling constants for these two groups seem to be about the same. Of course, we do not really know how to calculate these coupling constants, because only for very few of these weak interactions, like \(\beta\) decay and \(\mu\) decay, do we know something about the detailed dynamics of the decay reaction. In most of the decays we do not know how the various fields are coupled. Consequently, we can only estimate these coupling constants in a very crude way. For cases where the detailed interaction is unknown, we use the formula (with \(\hbar = c = 1\))

\[1/\tau = 2\pi G^2 (1/R)^2 \rho_\beta\]  

where \(\rho_\beta\) is the number of final states per unit energy,

\[\rho_\beta = \frac{8\pi^2}{\Omega} \int \prod_{i=1}^n \left( \frac{1}{8\pi^2} d^4p_i d^4r_i \right) \delta^4(\sum p_i) \delta(\sum E_i - E) ;\]
Table 2

<table>
<thead>
<tr>
<th>Reaction</th>
<th>Lifetime (sec)</th>
<th>$\mathcal{G}^2 \times 10^{14}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Lambda^0 \rightarrow p + \pi^-$</td>
<td>$3 \times 10^{-10}$</td>
<td>1.6</td>
</tr>
<tr>
<td>$\Sigma^- \rightarrow n + \pi$</td>
<td>$1.6 \times 10^{-10}$</td>
<td>1.2</td>
</tr>
<tr>
<td>$K^0 \rightarrow 2\pi$</td>
<td>$0.95 \times 10^{-10}$</td>
<td>2.8</td>
</tr>
<tr>
<td>$\pi^+ \rightarrow \mu^+ + \nu$</td>
<td>$2.6 \times 10^{-8}$</td>
<td>0.3</td>
</tr>
<tr>
<td>$\mu^+ \rightarrow e^+ + \nu + \bar{\nu}$</td>
<td>$2.2 \times 10^{-6}$</td>
<td>2</td>
</tr>
<tr>
<td>$K^- \rightarrow \mu^- + \nu$</td>
<td>$1.2 \times 10^{-8}$</td>
<td>0.02</td>
</tr>
</tbody>
</table>

$R$ represents a characteristic length of these decays; and $\Omega = (4\pi/3)R^3$. The total number of the particles in the decay product is $n$, and $p$, $E$, are their corresponding momenta and energies. In Table 2 are listed the various lifetimes and the corresponding coupling constants for several of the decay interactions in both group (i) and group (ii). In all these reactions we use

$$R = \hbar/m_\pi c$$  \hspace{1cm} (1.31)

for simplicity. These results of course have significance only in their crude order of magnitude. For the purpose of comparison we include in Table 2 also $\mu$ decay calculated in the same way, even though we do know a great deal about its coupling.\(^7\)

We observe from Table 2 the remarkable fact that although the lifetimes of these particles vary over a wide range from $10^{-10}$ sec to $10^{-6}$ sec the corresponding coupling constants $\mathcal{G}^2$ seem to be much more stable.

On the other hand, as remarked before, these decay interactions are separated into two distinct groups. In the first group, (i), the neutrinos are not involved; instead there is a nonconservation of $I_z$. In the other group, (ii), every reaction contains some neutrinos. Furthermore, these reactions are between many particles for which isotopic spin seems to play no role. The fact that they share approximately the same strength in coupling constants does suggest strongly a deep common origin for all weak interactions. As we shall see later, these weak interactions may share another significant feature, namely the violation of invariance under space inversion and charge conjugation.

\(^7\)The coupling constants for $\beta$ decay and $\mu$ capture are not included in Table 1. It is well known that they have the same order of magnitude as that for $\mu$ decay. See, e.g., E. Fermi, *Elementary Particles*, Yale University Press, 1951.
II. THE $\theta$-$\tau$ PUZZLE

Among the various interesting phenomena concerning elementary particles, we would like to discuss specifically first the $\theta$-$\tau$ puzzle, because it was due to this puzzle that a re-examination of the experimental basis of our various supposedly exact conservation laws was made. In Table 3 are listed the recently measured values of the mass, abundance, and lifetime of the various decay modes of charged $K$-mesons.\(^8\)

<table>
<thead>
<tr>
<th>Type</th>
<th>Abundance</th>
<th>from primary particle</th>
<th>from secondary particles</th>
<th>Lifetime</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau$</td>
<td>5.56±0.41</td>
<td>966.3±2.1</td>
<td>966.1±0.7</td>
<td>(1.19±0.05)$\times 10^{-8}$</td>
</tr>
<tr>
<td>$\tau'$</td>
<td>2.15±0.47</td>
<td>967.7±4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$K_{\pi^2}$</td>
<td>58.2±3.0</td>
<td>967.2±2.2</td>
<td>965.8±2.4</td>
<td>(1.24±0.02)$\times 10^{-8}$</td>
</tr>
<tr>
<td>$K_{\pi^2}$</td>
<td>28.9±2.7</td>
<td>966.7±2.0</td>
<td>962.8±1.8</td>
<td>(1.21±0.02)$\times 10^{-8}$</td>
</tr>
<tr>
<td>$K_{\pi^3}$</td>
<td>2.83±0.95</td>
<td>969±5</td>
<td></td>
<td>(0.88±0.23)$\times 10^{-8}$</td>
</tr>
<tr>
<td>$K_{\pi^3}$</td>
<td>3.23±1.30</td>
<td></td>
<td></td>
<td>(1.44±0.46)$\times 10^{-8}$</td>
</tr>
</tbody>
</table>

We see that the masses are extremely close to each other and the lifetimes agree within the experimental error of $\sim \pm 5\%$. About three years ago, Dalitz pointed out that by plotting the angular and energy distribution of the three $\pi$-mesons from the decay of the $\tau$-meson ($\equiv K_{\pi^3}$), it is possible to determine the spin and parity of the $\tau$. In the following discussion of the $\theta$-$\tau$ puzzle, we shall assume that both spin and parity are absolutely conserved.

1. Review of the Spin-Parity Determination of $\theta$ and $\tau$ (Dalitz's Analysis\(^9\))

Let us consider first a $\theta$-meson. The $\theta$-meson is defined to be a $K$-particle which can decay, among other modes, into two $\pi$-mesons, e.g.,

$$\theta^+ \rightarrow \pi^+ + \pi^0.$$  \hspace{1cm} (2.1)

Assuming that both the spin and parity are conserved in reaction (2.1), the parity


of the θ is uniquely determined by its spin value. Let $J$ be the spin of the θ-particle. Because each π-meson is a pseudoscalar, we have

$$P_\theta = (-1)^J$$  \hspace{1cm} (2.2)$$

where $P_\theta$ = parity of the θ-meson. Thus, the spin-parity assignment of θ can only be $(0+), (1-), (2+), $ etc.

On the other hand, the possible spin-parity assignments for a τ-meson are quite different. The τ-meson is defined to be a $K$-meson which can decay, among other modes, into three π-mesons, e.g.,

$$\tau^+ \rightarrow \pi^+ + \pi^+ + \pi^-.$$  \hspace{1cm} (2.3)$$

The final decay state of a τ-meson is characterized by two momenta in its center-of-mass system. We may choose these two momenta to be:

(i) The relative momentum, $p$, between the two $\pi^+$ mesons.

(ii) The momentum, $k$, of the $\pi^-$ in the center-of-mass system of the τ-meson. (It may differ by a factor 3/2, if the momentum is chosen to be that of the $\pi^-$ with respect to the center-of-mass system of the two $\pi^+$ system.)

Let $J$ be the spin of the τ-meson; then

$$J = L_p + L_k.$$  \hspace{1cm} (2.4)$$

where $L_p$ and $L_k$ are respectively the orbital angular momentum of the $\pi^-$ and the relative angular momentum of the two $\pi^+$. If the spin of the τ is zero, then the final state of the three pions must consist of states with $L_p = L_k$. Hence the parity of the final state must be $-1$. However, if the spin of τ is not zero then the parity of the final state can be either $+1$ or $-1$. Consequently the spin-parity assignments for τ are $(0-), (1\pm), (2\pm), $ etc.

It is easy to see the following simple conclusions:

(i) If the spin of τ is zero and if parity is conserved in the decay, then $\theta \neq \pi$.

(ii) If there exists a zero energy pion ($\pi^+$ or $\pi^-$) in the $3\pi$ state of τ decay and if parity is conserved in the decay, then $\theta \neq \pi$.

(iii) If there exists a zero energy $\pi^-$ in the $3\pi$ state of the τ decay then the spin of τ must be even. If, further, parity is conserved, the parity of τ must be odd.

Of course, in reality it is not possible to observe a zero energy pion. But quite a few low energy ($\sim 0.5$ Mev) $\pi^-$ and $\pi^+$ have been observed. Thus, even without detailed statistical analysis, it is to be expected that most probably the spin of τ is even, and that if parity is conserved in the decay process then τ and θ are two different particles. To evaluate the exact meaning of the likelihood it is necessary to perform a detailed statistical analysis.

Let us characterize the system by an angle $\theta$ and a parameter $\epsilon$ defined to be

$$\theta = \angle (p, k) \quad \text{and} \quad \epsilon = (k/k_{max})^2.$$  \hspace{1cm} (2.5)$$

If $\psi(p, k)$ represents the final state wave function of the $3\pi$ in the τ decay then the probability distribution function $P(\theta, \epsilon)$ is

$$P(\theta, \epsilon) \propto |\psi(p, k)|^2 \sqrt{\epsilon(1-\epsilon)} \, d\epsilon \, d(\cos\theta).$$  \hspace{1cm} (2.6)$$
Experimentally with an energy value ranging from \(\sim \frac{1}{2}\) Mev to \(\sim 50\) Mev and with a total of \(\sim 1000\) events, Dalitz and others find that

\[
|\psi|^2 = 1 \tag{2.7}
\]

is an extremely good fit to these data. From the above conclusion (ii) we see that if \(\tau\) is the same as \(\theta\), then \(|\psi|^2\) must be zero if, say, \(\epsilon = 0\). In fact, it is possible to prove the following rigorous statements concerning the behavior of \(\psi\) at various limiting regions.

(iv) If \(\tau\) is the same as \(\theta\) and if parity is conserved in the \(\tau\) decay, then

\[
as \epsilon \to 0, \ |\psi|^2 \to \epsilon^n \quad (n \geq 1) ;
\]

\[
as \epsilon \to 1, \ |\psi|^2 \to (1-\epsilon)^m \quad (m \geq 2) ;
\]

and as \(\theta \to 0, \ |\psi|^2 \to \sin^2 \theta . \tag{2.8}
\]

To prove (iv), we consider first the simple case that the spin of \(\tau\) is 1. From Equation (2.2) we see that if \(\tau\) is the same as \(\theta\) then the parity of \(\tau\) is \(-1\). Consequently the orbital parity due to \(k\) and \(p\) must be \(+1\). Thus, \(\psi\) should be an axial vector. Furthermore, \(\psi\) must be an even function of \(p\) because of the Bose-statistical property of the two \(\pi^-\)-mesons. Hence it is easy to deduce that

\[
\psi = (k \times p) (k \cdot p) f [k^2, (k \cdot p)^2, p^2] .
\]

Or,

\[
|\psi|^2 \propto \epsilon^2 (1-\epsilon)^2 \sin^2 \theta \cos^2 \theta |f|^2 . \tag{2.9}
\]

The function \(f\) is expected to be a regular function of its arguments. (This is the case, e.g., if the decay of \(\tau\) can be represented by a local field theory with decay interaction involving derivative couplings to any high, but finite, orders.) Hence we see that the wave function \(\psi\) satisfies the conditions given by Equation (2.8). Similarly, by going over the same type of arguments of forming various tensor functions with 2 vectors, \(k\) and \(p\), one can easily prove (2.8) for any other spin values.

From (2.9) we see that it is very difficult to pick a function \(f\) such that in all observed regions in \((\theta, \epsilon)\) space \(|\psi|^2 \approx 1\) so as to be compatible with experimental results. In fact, there are good arguments to expect the function \(f\) to be near 1. The reason is, as pointed out by Dalitz, that these pions are very low in energy, and an expansion of \(f\) into powers of \(k^2\) and \(p^2\) may be legitimate,

\[
f = 1 + 0(k^2 R^2) + 0(p^2 R^2) + \ldots . \tag{2.10}
\]

From dimensional grounds we expect \(R\) to be some length that characterizes the \(\tau\)-meson. For low energy pions we may neglect the terms \(0(k^2 R^2)\) and \(0(p^2 R^2)\) and we have

\[
f \approx 1 . \tag{2.11}
\]

By using the condition \(f \approx 1\) in the distribution function \(\psi\), we can calculate the probability that one obtains \(|\psi|^2 = 1\) from experimental events if \(\psi\) is actually given
by Equation (2.9). This probability will be extremely small ($\lesssim 10^{-30}$). Similar conclusions can be obtained for other spin values (except for very high ones). Thus, we conclude that it is extremely improbable that $\tau$ and $\theta$ are the same particle (under, of course, the framework that parity is conserved). The most probable spin-parity assignments of $\tau$ are $0-$ and $2-.$

One may question how good is the approximation $f \approx 1.$ The average kinetic energy of $\pi$ in the $\tau$ decay is $\sim 26$ Mev. If $R$ is taken to be $\hbar/m_{\pi}c$ where $m_{\pi}$ is the pion mass, then

$$(k^2R^2) \approx \frac{1}{3},$$

which is not very small. However the region where the distribution function $|\psi|^2$ [Equation (2.8)] deviates most from the experimental situation is precisely the region (e.g., $k \to 0$ or $p \to 0$) where the relevant expansion parameter is small. Consequently, one expects that the conclusions of Dalitz are statistically very significant. Thus, there seem to be two particles of different spin-parity values. The difficulty is, then, why should they have, within experimental error, the same mass and the same lifetime? This is the famous $\theta$-$\tau$ puzzle.

2. Previous Attempts to Solve the $\theta$-$\tau$ Puzzle

Mainly for historical reasons we shall review some attempts that have been made to solve the $\theta$-$\tau$ puzzle. Although the Dalitz analysis was made more than three years ago, at that time various different masses of $K$-mesons were reported. In fact there were indications that there might be a big mass difference between the various $K$-particles. Also, the statistics of Dalitz's analysis at that time were not very good. No one was greatly alarmed that there probably existed a $K$-particle ($\tau$-meson) which was different from the $\theta$-particle. There were too many $K$-particles anyway. However, about the spring of 1955, the situation was changed. The experimental mass values gradually converged, though still with large probable error. The Dalitz plot had more points and seemed to indicate convincingly that the $\tau$ is a $(0-)$-particle. Also, at about the same time people started to measure the lifetime. Before the lifetime measurements were done, some physicists speculated about what would happen if the lifetimes turned out to be the same. At first sight it seemed that, if the lifetimes of $K_{\pi 2}$ and $K_{\pi 3}$ are measured to be the same, then this evidence would be used to argue that $\theta$ must be the same particle as $\tau$ and that the conclusion of Dalitz's analysis is probably a manifestation of some statistical fluctuations. Later the experiments on lifetimes indeed showed that the observed decay constants for $K_{\pi 2}$ and $K_{\pi 3}$ are about the same.

Nevertheless, with all the available evidence at that time (1955) it was not difficult to find schemes which could be made compatible with (i) the results of Dalitz's analysis, (ii) the apparent identity of lifetimes, and (iii) the approximate equality of masses. In describing these attempts we shall assume that the conclusions concerning Dalitz's analysis are correct and that $\tau$ and $\theta$ are two different particles.

---

A. APPARENT LIFETIME EQUALITY

One such hypothesis, proposed to explain the apparent identity in lifetimes, is the so-called cascade process.\(^{11}\) The idea is that \(\tau\) and \(\theta\) are two different particles with two different lifetimes, say, \(10^{-8}\) sec and \(10^{-10}\) sec. (On phase space arguments, one expects \(K_{\pi^{0}}\) to live longer than \(K_{\pi^{+}}\).) The long-lived one, say \(\tau\), is assumed to have a heavier mass. In addition to its other decay modes, \(\tau\) decays into the light one, \(\theta\), by \(\gamma\) radiations. Under the experimental conditions in these lifetime measurements, only the long-lived \(K\)-mesons would be observed. Thus this cascade process can account for their apparent equality of lifetimes. If the spin-parities of \(\tau\) and \(\theta\) are \(0-\) and \(0+\), then the cascade process is

\[\tau \rightarrow \theta + 2\gamma.\]  

(2.11)

In order to make the branching ratio correct a mass difference \(m_{\tau} - m_{\theta} \cong 10\) Mev is required.

B. MASS DEGENERACY\(^{12}\)

By taking analogy with isotopic spin invariance, a mass degeneracy means possibly a new symmetry property. This, for example, is the case of mass degeneracy between a proton and a neutron. The only difference is that we now have degeneracy between states with different parities, instead of between states with different charges. If one regards this mass degeneracy between \(\tau\) and \(\theta\) to be not an accident, then it means, just as in the case for isotopic spin, that the strong interaction must be invariant under a new symmetry operator. This operator, denoted by \(C_{p}\), when acting on a \(\theta\)-particle changes it into a \(\tau\), and when acting on \(\tau\) converts it into a \(\theta\). The operator \(C_{p}\) is called “parity conjugation” by analogy with “charge conjugation.”

\[C_{p}\theta = |\tau\rangle, \quad C_{p}\tau = |\theta\rangle.\]  

(2.12)

The approximate mass degeneracy now follows from the assumption that \(C_{p}\) commutes with the strong part of the Hamiltonian

\[[C_{p}, H_{\text{strong}}] = 0.\]  

(2.13)

Because of Equation (2.13) it follows that all strong interactions should be invariant under the operation of \(C_{p}\). In particular, let us take an example, say,

\[\pi^{+} + p \rightarrow \Lambda^{0} + \theta^{0}.\]  

(2.14)

Under the operation \(C_{p}\), there is no change in \(\pi^{+} + p\) but \(\theta^{0}\) becomes \(\tau^{0}\). Therefore the \(\Lambda^{0}\) must be a parity doublet, which we shall denote by \(\Lambda_{1}^{0}\) and \(\Lambda_{2}^{0}\). Equation (2.14) becomes

\[\pi^{+} + p \rightarrow \Lambda_{2}^{0} + \tau^{0}\]  

(2.15)

under \(C_{p}\).

We conclude therefore that there must be a parity doubling not only of the \(K\)-mesons but also of hyperons. There will be two \(\Lambda\)'s of opposite parity and two \(\Sigma^{+}\)'s


of opposite parity, etc. In fact, there must be a parity doublet for all strange particles
with odd strangeness quantum number. The commutation relation between \( C_p \) and
the \( H_\gamma \) (electromagnetic interaction) is not known. A natural choice, of course, is to
take advantage of the above explanation for the lifetimes and to assume that \( C_p \) does
not commute with \( H_\gamma \). This can introduce a large mass difference and make the
cascade process possible. Combining these two possibilities, it seemed that one could
have an explanation for the \( \theta-\tau \) puzzle. However, within half a year,\(^8\) the mass measure-
ments have been greatly improved, with the result that the mass difference can
at most be \( \sim 1 \) or 2 Mev, which makes process (2.11) very unlikely. A direct search
for such \( \gamma \)-rays was performed\(^{13}\) and it also led to negative results.

In the early part of 1956 it seemed that the true sloution of the \( \theta-\tau \) puzzle might
in fact lie in something quite different. Thus, an investigation of the experimental
basis of the law of conservation of parity was made.\(^{14}\) We shall discuss now in some
detail the conclusions reached through such an investigation.

III. EXPERIMENTAL LIMITS ON THE VALIDITY OF PARITY CONSERVATION

In this section we shall try to discuss the limit on the validity of parity conservation in various fields of physics. If parity is not a rigorously conserved quantum number then eigenstates $\psi$ of the entire Hamiltonian are, in general, not eigenstates of the parity operator. Thus we expect

$$\psi = \psi_p + F \psi_{-p}$$  \hspace{1cm} (3.1)

where $\psi_p$ and $\psi_{-p}$ are of opposite parity and

$$F = \text{probability amplitude for parity mixing.}$$  \hspace{1cm} (3.2)

It is useful to know from the various evidence in atomic and nuclear physics exactly what is the upper limit one can impose on the magnitude of $F$.

1. Atomic Spectroscopy

From the various parity selection rules concerning the radiative transitions for an atomic system, we find an upper limit for $F$:

$$|F|_{\text{atom}} \leq \left( \frac{r}{\lambda} \right)^2_{\text{atom}} \approx 10^{-6}$$  \hspace{1cm} (3.3)

for a typical atomic transition. In (3.3), $r$ is the radius of the atom and $\lambda$ the wavelength of the radiation. In principle, by studying transitions involving photons of long wavelengths it is possible to make this upper limit much smaller than $10^{-6}$. (It may not be impossible to reach the limit $|F|_{\text{atom}} < 10^{-12}$.)

2. Nuclear Spectroscopy

While the above condition sets a limit on parity nonconservation in atomic interactions, the same limit cannot be applied directly to nuclear interactions. Nevertheless, by using the various parity selection rules in nuclear spectroscopy, say $\beta$ decay, it is possible to put a corresponding limit for the nuclear system,

$$|F|_{\text{nucleus}}^2 \leq \left( \frac{r}{\lambda} \right)^2_{\text{nucleus}} \approx 10^{-4}.$$  \hspace{1cm} (3.4)

3. Nuclear Reactions

The measurement by Chamberlain et al.\textsuperscript{15} on the double scattering of protons offers a very direct test of parity conservation. In this experiment, a beam of incom-

ing protons with momentum $p_1$ is scattered by a first target into momentum $p_2$ and is further scattered by a second target into momentum $p_n$. If parity is conserved, the cross section should be independent of such a quantity as $(p_1 \times p_2) \cdot p_n$. The measurement shows the absence of such terms, giving directly an upper limit on $F$. From the measurements one can conclude

$$|F|^2 < 10^{-14}. \quad (3.5)$$

4. Static Electric Dipole Moment

If parity is conserved then the static electric dipole moment of any eigenstate of the Hamiltonian must be zero. Thus a measurement on the absence of such electric dipole moment for elementary particles gives also an upper limit of $F$. Smith, Purcell, and Ramsey\(^{16}\) have measured the electric dipole moment of the neutron and found it to be smaller than $5 \times 10^{-20} \text{ cm} \times \text{ electronic charge}$. If one takes the natural size of the neutron to be $10^{-13} \text{ cm}$, this gives a very severe limit on $F$:

$$|F|^2 < 3 \times 10^{-12}. \quad (3.6)$$

This limit applies directly to the structure of the neutron.

However, it is possible to show that if time-reversal invariance holds then the static electric dipole moment must still be zero even though parity may not be conserved. This is so because if parity is not conserved then the wave function $\psi$ is a mixture of states with opposite parities $\psi_p$ and $\psi_{-p}$ as indicated by Equation (3.1). But if time reversal is invariant then $\psi_p$ and $\psi_{-p}$ will be $90^\circ$ out of phase. Thus they cannot contribute to the diagonal element of the electric dipole moment which is a real quantity. We shall give a formal proof of the impossibility of having a static electric dipole if time reversal is invariant.

Consider a particle $A$ with spin $J$. The state function $|A\rangle_m$ describes the particle $A$ at rest with $J_z = m$. The time reversal operator $T$ is represented by

$$T = U_T \times \text{complex conjugation} \quad (3.7)$$

where $U_T$ is a unitary operator. If invariance under time reversal is assumed, then

$$T |A\rangle_m = U_T |A^*\rangle_m = e^{i\delta_m} |A\rangle_{-m} \quad (3.8)$$

where $^*$ represents a pure complex conjugation, and $e^{i\delta_m}$ is a possible phase factor. Let $D$ be the electric dipole moment,

$$D = \sum e_i r_i \quad (3.9)$$

The average value of $D$ must be proportional to that of $J$.

$$\langle A|D|A\rangle_m = K \langle A|J|A\rangle_m \quad (3.10)$$

where $K$ is a real numerical constant. If we take the complex conjugation of both sides in Equation (3.10) and replace $1$ by $U_T^* U_T$, Equation (3.10) becomes

\[
(A^*|U^*_r D^* U^*_r U^*_r |A^*)_m = K^* (A^*|U^*_r J^* U^*_r U^*_r |A^*)_m.
\]

From (3.8) and the properties that
\[
TJ T^{-1} = U^*_r J^* U^*_r = -J \quad \text{and} \quad Tr T^{-1} = U^*_r r^* U^*_r = +r,
\]
we have
\[
(A|D|A)_m = - K (A|J|A)_m.
\]

Comparison of (3.10) and (3.12) shows that
\[
(A|D|A)_m = 0.
\]

Thus, the static electric dipole moment must be zero if time reversal is invariant. However, the matrix elements of \( r \) between two different states of dominantly the same parity are, in general, not zero if time reversal invariance holds but parity is not conserved. (Here the term "same parity" refers to that part determined by the strong interactions.)

Thus, if time reversal invariance holds, the most severe limit on parity nonconservation is given by spectroscopic evidence and experiments of the nuclear double scattering type. These limits are already quite strong and thus demand that the strong interactions and the electromagnetic interactions both must conserve parity. However, these limits throw no light on the invariance properties of the weak interactions.

5. \( \beta \) Decay Experiments

Prior to the recent experiments on \( \beta \) angular distribution from polarized nuclei and on the longitudinal polarization of \( \beta \)-rays, there already existed an immense body of knowledge in the field of \( \beta \) decay. These previous experiments consist of (i) spectra (allowed, forbidden, etc.) and \( \beta \) values, (ii) \( \beta \)-neutrino correlation, (iii) \( \beta \)-\( \gamma \) correlation, (iv) polarized nuclei and the angular distribution of secondary \( \gamma \)-rays, and (v) \( \beta \)-\( \gamma \) angular correlation. We shall show that these experiments (i) to (v) are not relevant so far as the question of parity conservation in weak interactions is concerned. They neither prove nor disprove the conservation of parity in \( \beta \) decay.

The most general form of the interaction Hamiltonian for nonconservation of parity is
\[
H_{\text{int}} = \sum_\sigma (\bar{\psi}_p O_\sigma \psi_n) (C_\sigma \bar{\psi}_p \psi_n + C_\sigma' \bar{\psi}_p' \psi_n' γ_5 \psi_n'), \quad i = S, T, V, A, P, \tag{3.14}
\]
where \( O_\sigma = γ_\sigma, O_v = γ_4 γ_μ, O_T = -i(2/\sqrt{2}) γ_4(γ_λ γ_μ - γ_μ γ_λ), O_A = -iγ_4 γ_μ γ_5, \) and \( O_P = γ_4 γ_5 \). In Equation (3.14) derivative couplings are not included. The conclusions [Equations (3.18) and (3.20) below] are correct even if there are such derivative coupling terms. We have now ten complex coupling constants. Any observed quantity will be related to the sum of the absolute squares of certain matrix elements.

\[\text{---}
\]
\[\text{---}\]

\[\text{---}\]

\[\text{---}\]
\[ \sum |M|^2 = (\sum f_{ij} C_i^* C_j + \text{c.c.}) + (\sum f_{ij}' C_i^* C_j' + \text{c.c.}) + (\sum g_{ij} C_i^* C_j' + \text{c.c.}) \]  
(3.15)

where \( f_{ij}, f_{ij}' \) and \( g_{ij} \) are certain functions of the measured momenta and spins. It is well known that, as the neutrino has zero mass, it satisfies not only the familiar Dirac equation

\[ \left( \gamma_\mu \frac{\partial}{\partial x_\mu} \right) \psi = 0 , \]  
(3.16)

but also the equation

\[ \gamma_\mu \left( \frac{\partial}{\partial x_\mu} \right) \gamma_5 \psi = 0 . \]  
(3.17)

Thus, one has

\[ f_{ij} = f_{ij}' . \]  
(3.18)

Furthermore, we can show that the \( g_{ij} \) must be pseudoscalar quantities. To see this, let us consider the following formal transformation:

\[ C_i \rightarrow C_i' , \quad C_i' \rightarrow -C_i' \]  
(3.19)

together with \( r \rightarrow -r; \quad p \rightarrow -p; \) and spin \( s \rightarrow +s . \) This formal mathematical transformation leaves the Hamiltonian \( H_{1\text{nt}} \) invariant. Thus it must also leave Equation (3.15) invariant. It then follows that under this formal transformation the interference terms \( g_{ij} \) must transform as

\[ g_{ij}(p, s, \ldots ) \rightarrow g_{ij}(-p, +s, \ldots ) = -g_{ij}(p, s, \ldots ) . \]  
(3.20)

Consequently \( g_{ij} \) transform like pseudoscalar quantities. This means that in order to answer the question of parity conservation it is necessary to observe, experimentally, a pseudoscalar quantity. To observe a pseudoscalar quantity one must measure at least three linear momenta \( p_1, p_2, p_3 \) or a spin \( s \) and a momentum \( p \) so as to form quantities like \( (p_1 \times p_2) \cdot p_3 \) or \( s \cdot p \), etc. In the experiments on the spectra, \( \beta-\nu \) correlation and \( \beta-\gamma \) correlation, it is clear that no such pseudoscalar quantity can be formed. With the parity nonconserving Hamiltonian Equation (3.14), the theoretical results for these experiments are identical with that of the conventional parity conserving Hamiltonian provided one replaces \( C_i^* C_j \) in the old formulas:

\[ C_i^* C_j \rightarrow (C_i^* C_j + C_i^* C_j') . \]  
(3.21)

In the measurement of polarized nuclei and the angular distribution of the secondary \( \gamma \)-ray it is possible to form pseudoscalar terms like

\[ (s \cdot p_\gamma) . \]  
(3.22)

However, since \( \gamma \) interaction conserves parity and since the multipole \( \gamma \) radiation in nuclear transitions has very accurately defined parities, the observed angular distribution must be invariant under the transformation

\[ \text{It is important to note that in general (3.15) is invariant under the mathematical transformation } C_i \rightarrow C_i' \text{ and } C_i' \rightarrow C_i. \text{ This property can serve as a good check for the correctness of various expressions [cf. Equations (5.3), (5.11), etc.]. We wish to thank Dr. Pauli for a communication on the usefulness of this transformation.} \]
\[ p_\gamma \rightarrow -p_\gamma. \]  \hspace{1cm} (3.23)

Thus terms of the form (3.22) cannot exist.\(^{20}\)

In the case of \(\beta\)-\(\gamma\)-\(\gamma\) angular correlation measurements, one can easily form pseudoscalar quantities that are also invariant under transformation (3.23), such as

\[ P_e \cdot (p_\gamma \times p_{\gamma'}) (p_\gamma \cdot p_{\gamma'}). \]  \hspace{1cm} (3.24)

These terms cannot be ruled out by using the parity conservation property of the \(\gamma\) radiation. However, if time reversal is invariant for the strong interactions (including \(\gamma\) interactions), then such terms must all vanish. [This follows immediately from Equation (4.29)]. Consequently the absence of such terms can be used to prove the invariance of time reversal for strong interactions and electromagnetic interactions but not for the invariance properties of the weak interactions. We can summarize as follows: The previous accurate measurements of \(\beta\) decay (i) to (v) do not throw any light whatsoever upon a possible nonconservation of parity in the decay process. In order to detect possible parity nonconservation terms we must try to perform other experiments such as to measure \(s \cdot p_e\), etc.

\(^{20}\)The nonexistence of terms like \(s \cdot p_\gamma\) and \(p_e \cdot (p_\gamma \times p_{\gamma'})\) can also serve as evidence for the parity conservation for the strong interactions (including \(\gamma\) interactions) in a nuclear system, but not for weak interactions.
IV. SOME GENERAL DISCUSSIONS ON THE CONSEQUENCES DUE TO POSSIBLE NON-INVARiance UNDER $P$, $C$, AND $T$

1. The CPT Theorem.\textsuperscript{21} Equality of Mass and Lifetime Between a Particle and its Antiparticle\textsuperscript{22}

Before we discuss in detail the various tests on the conservation of parity $P$, charge conjugation $C$, and time reversal $T$ in weak interactions, it is useful to recall a general theorem concerning the interrelationship between these operators $C$, $P$, $T$ and proper Lorentz invariance.

The CPT Theorem: If a local Lagrangian theory (which may contain derivative couplings to any high but finite orders) is invariant under the proper Lorentz transformations, it is invariant under the product of $CPT$ (and its permutation $PCT$, etc.) although the theory may not be separately invariant under each one of these three operators $C$, $P$, and $T$.

It follows from this theorem that if $P$ is not conserved in the weak interactions, then at least one of the other invariances $C$ or $T$ should not be conserved. In the following discussions we shall assume that the general framework of field theory, under which the CPT theorem is proved, is valid. At first sight it seems that the observed equality of lifetimes in the decay of $\pi^+$, $\pi^-$ and the similar equality for the $\mu^+$, $\mu^-$ may already form a proof that $C$ is conserved in weak decays. As we shall see, the equality of the masses and lifetimes for a particle and its antiparticle follows directly from proper Lorentz invariance and the CPT theorem. It does not prove at all that $C$ is invariant. We shall state these consequences of the CPT theorem in the form of two theorems. Let $H$ be the complete Hamiltonian which may be separated into two parts

$$H = H_s + H_w \quad (4.1)$$

where $H_s$ represents the strong interactions together with the $\gamma$ interactions, and $H_w$ the weak interactions. We assume that both $H_s$ and $H_w$ are invariant under the proper Lorentz transformation. Consequently, $H_s$ and $H_w$ are both invariant under the compound operation of $PCT$, i.e.,

$$PCTHT^{-1}C^{-1}P^{-1} = PCU_TH^*U_T^*C^*P^* = H \quad (4.2)$$

where $P$, $C$, $U_T$ are all unitary operators [cf. Equation (3.7)]. We shall further assume that $H_s$ is invariant under the separate operation of each one of these three operators $C$, $P$, and $T$ while $H_w$ may or may not be invariant under $C$, $P$, and $T$ separately. (The operators $C$, $P$, $T$ are defined by using the invariance properties of $H_s$.)


Theorem 1: If $\alpha$ is a stable particle, then
\[ M_\alpha = M_{\bar{\alpha}} \]  \hspace{1cm} (4.3)
where $M_\alpha$ and $M_{\bar{\alpha}}$ are the masses of $\alpha$ and its antiparticle $\bar{\alpha}$. Equation (4.3) is valid to all orders in $H_\omega$.

Proof: Consider a particle $\alpha$ at rest.
\[ H|\alpha\rangle = M_\alpha|\alpha\rangle \quad \text{or} \quad H^*|\alpha^\ast\rangle = M_\alpha|\alpha^\ast\rangle \]  \hspace{1cm} (4.4)
and
\[ PCU_T H^*|\alpha^\ast\rangle = M_\alpha \cdot PCU_T |\alpha^\ast\rangle . \]  \hspace{1cm} (4.5)
Let
\[ |\bar{\alpha}\rangle \equiv PCU_T |\alpha^\ast\rangle . \]  \hspace{1cm} (4.6)
We then have
\[ H|\bar{\alpha}\rangle = M_\alpha |\bar{\alpha}\rangle \]  \hspace{1cm} (4.7)
by using Equation (4.2). From the definitions of $P$, $C$, $T$ [see Chapter I, section 3, and Equation (3.8)] we know that if $|\alpha\rangle$ represents a particle at rest with spin $J$ and its $z$ component $J_z$, then $|\alpha\rangle$, defined by Equation (4.6), represents its antiparticle state also at rest and with its spin along $z$ component $-J_z$. Theorem 1 follows immediately from Equation (4.7).

Remarks: What we have proved is actually more than just the equality of masses. By taking $|\alpha\rangle$ to be any eigenstate of the total Hamiltonian $H$ we can generate another state $|\bar{\alpha}\rangle$ by (4.6) which has the same eigenvalue. Thus the complete energy spectrum for a group of particles is identical with that of a corresponding group of antiparticles. By considering the energy spectrum of a particle, say a proton $p$, in a magnetic field, one can prove the equality of the magnitude of magnetic moment of $p$ and $\bar{p}$ (again to all orders of $H_\omega$).

Theorem 2: Consider the decay of $A$ via $H_\omega$,
\[ A \rightarrow B \quad \text{and} \quad \bar{A} \rightarrow \bar{B} \]  \hspace{1cm} (4.8)
(with the states $B \neq \bar{B}$);\(^{24}\) then, to the lowest order in $H_\omega$,
\[ \text{(lifetime of } A) = \text{(lifetime of } \bar{A}). \]  \hspace{1cm} (4.9)

Proof: Since we are only interested in the lowest order in $H_\omega$, the states $|A\rangle_m$, $|B\rangle_m$, $|\bar{A}\rangle_m$, $|\bar{B}\rangle_m$ can be taken to be eigenstates of $H_\omega$ with $z$ component spin $J_z = m$. Furthermore, since $H_\omega$ is invariant under $T$ and $C$, we have [cf. Equation (3.8)]
\[ |\bar{A}\rangle_m = C|A\rangle_m , \quad |\bar{B}\rangle_m = C|B\rangle_m ; \]  \hspace{1cm} (4.10)
\[ T|A\rangle_m = e^{i\delta_m (A)}|A\rangle_{-m} , \quad T|B\rangle_m = e^{i\delta_m (B)}|B\rangle_{-m} ; \]  \hspace{1cm} (4.11)
and similar equations for $|\bar{A}\rangle_m$ and $|\bar{B}\rangle_m$ under time reversal. The states $|B\rangle_m$ and $|\bar{B}\rangle_m$ are taken to be stationary states consisting of standing waves (including all the strong interactions).

\(^{24}\)This condition $B \neq \bar{B}$ applies to cases, e.g., where $A$ and $\bar{A}$ have opposite charges $\pm Q$ or opposite heavy particle number $\pm N$, etc. General discussions concerning cases where $\bar{B}$ may be the same as $\bar{B}$ (e.g., in the decay of $K^0$ and $\bar{K}^0$) will be found in Chapter V, section 9.
We separate
\[ H_w = H_+ + H_- \quad \text{with} \quad H_\pm \equiv \frac{1}{2} [ H_w \pm PH_w P^t ] \]
\[(4.12)\]

Hence
\[ PH_\pm P^t = \pm H_\pm \] \[(4.13)\]

The matrix elements \( \langle B | H_z | A \rangle \) are related to \( \langle \bar{B} | H_\pm | \bar{A} \rangle \) by the CPT theorem,
\[
\langle B | H_z | A \rangle_{m^*} = \langle B | T^{-1} H_z T^{-1} T | A \rangle_m
\]
\[
= \langle B | T H_z T^{-1} | A \rangle_{-m} e^{i\theta} = \langle B | C^* P^t H_\pm P C | A \rangle_{-m} e^{i\theta} = \pm \langle \bar{B} | H_\pm | \bar{A} \rangle_{-m} e^{i\theta} \quad (4.14)
\]

where \( \theta = \delta_m(A) - \delta_m(B) \). In the expression for the lifetime of \( A \), no pseudoscalar quantities can be formed. From arguments similar to those used in the previous chapter, we conclude that there can be no interference term between \( H_+ \) and \( H_- \). Thus we have
\[
\text{(lifetime of } A \text{)} = \sum_B (| \langle B | H_+ | A \rangle_m|^2 + | \langle B | H_- | A \rangle_m|^2) \cdot (\text{phase space})_B. \quad (4.15)
\]

Equation (4.15) is clearly independent of the \( m \) value. From (4.14) we have
\[
\text{(lifetime of } A \text{)} = \text{(lifetime of } \bar{A} \text{)}.
\]

From theorems 1 and 2, we conclude that the equality of the masses and lifetimes of a particle and its antiparticle cannot be used as evidence for the invariance under the charge conjugation operator \( C \). Rather, it may serve as evidence for the validity of the CPT theorem. Indeed, as we shall see later, the operator \( C \) is not conserved, at least in some of the weak interactions.

2. General Remarks Concerning Invariance or Non-invariance Under Time Reversal

Consider the weak decay of a particle
\[
A \rightarrow B \quad (4.16)
\]
through the weak interaction \( H_w \). Let \( H_w \) be written as a sum of many terms (with \( C_i \) as coupling constants),
\[
H_w = \sum C_i H_i, \quad (4.17)
\]
such that under a time reversal operation \( T \)
\[
T H_w T^{-1} = \sum C_i^* H_i \quad \text{and} \quad T H_i T^{-1} = H_i \quad (4.18)
\]
Thus, if \( T \) is invariant then \( C_i \) are all real and vice versa. The proof or disproof concerning the invariance under time reversal, then, rests completely on the possibility of measuring the relative phases between these \( C_i \). We consider first case A.
A. CASE IN WHICH THERE ARE NO STRONG INTERACTIONS BETWEEN VARIOUS DECAY PRODUCTS IN THE FINAL STATES

In this case the final state is given as

$$\psi = \sum_{p,s} \langle p, s | H_{ef} | A \rangle \cdot | p, s \rangle$$

(4.19)

where $| p, s \rangle$ represents free particle states of momenta $p_1, p_2 \ldots$ and spins $s_1, s_2 \ldots$ in the final state $B$. By assumption $| p, s \rangle$ is an eigenstate of $H_z$. From (4.17) we can write

$$\psi = \sum C_i M_i(p, s) \cdot | p, s \rangle$$

(4.20)

where

$$M_i(p, s) \equiv \langle p, s | H_z | A \rangle.$$  

(4.21)

Using (4.18) and the property [cf. Equation (3.11)]

$$T \{ p, s \} = \{-p, -s\},$$

(4.22)

we have

$$M_i^*(p, s) = M_i(-p, -s).$$

(4.23)

Now let us consider the measurement of an observable $O$ which is a function of the momenta $p_i$ and spins $s_i$ of some of the particles in the final state. Using (4.20), we obtain

$$\langle \psi | O | \psi \rangle = \sum_{i,j} C_i^* C_j O_{ij}$$

(4.24)

where

$$O_{ij} \equiv \sum_{p,s} M_i^*(p, s) M_j(p, s) \{ p, s | O | p, s \}.$$  

(4.25)

It is clear that

$$O_{ij}^* = O_{ji}.$$  

(4.26)

Let us separate various observables $O$ into even and odd functions $p$ and $s$ (see Table 4). We denote by $O_+$ the even functions and $O_-$ the odd functions. Hence

$$\{ p, s | O_+ | p, s \} = \pm \{ -p, -s | O_+ | -p, -s \},$$

(4.27)

and

$$T O_+ T^{-1} = \pm O_+.$$  

(4.28)

Using (4.25), (4.22), and (4.27) we have

$$(O_+)_ij = (O_+)_{ji} = \text{real} \quad \text{and} \quad (O_-)_ij = -(O_-)_{ji} = \text{imaginary}.$$  

(4.29)

<table>
<thead>
<tr>
<th>Table 4</th>
</tr>
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<tbody>
<tr>
<td>Some Examples of $O_+$ and $O_-$</td>
</tr>
<tr>
<td>$p_1 \cdot p_2$</td>
</tr>
<tr>
<td>$s \cdot p$</td>
</tr>
<tr>
<td>$s_1 \cdot s_2$</td>
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Thus, the diagonal elements of $O_-$ are related to the real and imaginary parts of $C_i^*C_j$, i.e.,

$$\langle \psi | O_- | \psi \rangle = \sum_{i,j} (O_-)_{i,j} (C_i^*C_j + C_iC_j^*) \quad \text{and} \quad \langle \psi | O_- | \psi \rangle = \sum_{i,j} (O_-)_{i,j} (C_i^*C_j - C_iC_j^*) \ .$$

Consequently, we reach the conclusion that if in the final state there are no strong interactions between the various decay products then the existence of any observables of the form $O_-$ serves as a proof that $H_w$ is not invariant under time reversal.

Next we consider case B.

**B. CASE IN WHICH THERE ARE STRONG INTERACTIONS BETWEEN VARIOUS DECAY PRODUCTS IN THE FINAL STATE**

In this case $|p, s\rangle$ is not an eigenstate of the strong Hamiltonian $H_s$. Let $|B\rangle$ be an eigenstate of $H_s$. Furthermore state $|B\rangle$ is chosen to be a stationary state. Since $H_s$ is invariant under $T$, we have [cf. Equation (3.8)]

$$T |B\rangle_m = |B\rangle_{-m} \quad (4.30)$$

where $J_z = m$. For simplicity we have absorbed the phase $e^{i\eta_B}$ in the definition of $|B\rangle_{-m}$. The final state in the momentum representation is now

$$|\psi\rangle = \sum_{B, p, s} \langle p, s | B^{out} \rangle \langle B | H_w | A \rangle \cdot |p, s\rangle \quad (4.31)$$

where the sum also extends over various final states $B$. The state $|B^{out}\rangle$ is the outgoing wave part of the stationary states $|B\rangle$. The state $|B^{out}\rangle$ contains a phase factor $e^{i\eta_B}$ with $\eta_B$ as the phase shift due to the final state interactions. Thus, by using (4.30), (4.22), and (4.18) we can write Equation (4.31) as

$$|\psi\rangle = \sum C_i e^{i\eta_B} M_{iB}(p, s) \cdot |p, s\rangle \quad (4.32)$$

with

$$M_{iB}^*(p, s) = M_{iB}(-p, -s) \ . \quad (4.33)$$

Similar to (4.24), we have

$$\langle \psi | O_+ | \psi \rangle = \sum C_i^* C_j e^{i\eta_B} O_{iB, jB} \quad (4.34)$$

and

$$O_{iB, jB} = \sum_{p, s} M_{iB}^*(p, s) M_{jB}(p, s) \langle p, s | O_+ | p, s \rangle \ . \quad (4.35)$$

Again, as in case A, we separate $O$ into two different types, $O_+$ and $O_-$. Similar to (4.29), their matrix elements are respectively

$$\langle \psi | O_+ | \psi \rangle = \sum_{iB, jB} [(C_i^* C_j + C_i C_j^*) \cos(\eta_B - \eta_{B^*}) +$$

$$i(C_i^* C_j - C_i C_j^*) \sin(\eta_B - \eta_{B^*})] \quad (4.36)$$

and

$$\langle \psi | O_- | \psi \rangle = \sum_{iB, jB} [(C_i^* C_j - C_i C_j^*) \cos(\eta_B - \eta_{B^*}) +$$

$$i(C_i^* C_j + C_i C_j^*) \sin(\eta_B - \eta_{B^*})] \ . \quad (4.37)$$

Also, as in the case of (4.28) we can prove
\[(O_{+})_{iB_{j}B'} = \text{real} \quad \text{and} \quad (O_{-})_{iB_{j}B'} = \text{imaginary}. \tag{4.38}\]

It is easy to see that if there are no final state interactions \((\eta_{B} = \eta_{B'} = \ldots = 0)\) then Equation (4.37) reduces to (4.29). In the present case measurements on both \(O_{+}\) and \(O_{-}\) may serve as tests on time reversal. The terms \((C_{i}^{*}C_{j} - C_{j}^{*}C_{i}) \sin(\eta_{B} - \eta_{B'})\) in \(O_{+}\) and the terms \((C_{i}^{*}C_{j} - C_{j}^{*}C_{i}) \cos(\eta_{B} - \eta_{B'})\) in \(O_{-}\) can be used to detect possible violation of time reversal invariance. As examples we list, for \(\beta\) decay, terms of the form \((p_{e} \cdot p_{p})Z\), \((s \cdot p_{e})Z\), and \(s \cdot (p_{e} \times p_{p})\). (For detailed discussions see Chapter V, sections 1 to 3.)

3. **Remark on Invariance Under Charge Conjugation**

In this section we shall make only a general remark concerning invariance under charge conjugation. If we separate \(H_{\omega}\) into the parity conserving part \(H_{1}\) and the parity nonconserving part \(H_{2}\) [cf. Equation (4.12)],

\[H_{\omega} = H_{1} + H_{2}\]

with

\[PH_{1}P^{\dagger} = H_{1} \quad \text{and} \quad PH_{2}P^{\dagger} = -H_{2}. \tag{4.40}\]

Furthermore, we decompose \(H_{1}\) and \(H_{2}\) into the sum of various terms as in Equation (4.17),

\[H_{1} = \sum_{i}(C_{1})_{i}(H_{1})_{i}, \quad H_{2} = \sum_{i}(C_{2})_{i}(H_{2})_{i}\]

with

\[T(H_{\alpha})_{i}T^{-1} = (H_{\alpha})_{i} \quad (\alpha = 1, 2). \tag{4.42}\]

Thus, if \(T\) is invariant, \((C_{1})_{i}\) and \((C_{2})_{i}\) must all be real. From the CPT theorem it follows now that if charge conjugation \(C\) is conserved then \((C_{1})_{i}\) must be real while \((C_{2})_{i}\) must be imaginary.

4. **Summary**

We summarize the above discussions [cf. Equations (4.36) and (4.37)].

(i) Let \(O_{+}\) be a scalar and be even in \(p\) and \(s\) (e.g., \(p_{1} \cdot p_{2}\)). The observation of such a term in the form

\[(O_{+})_{\text{scalar}} = \sum O_{iB_{j}B'}(C_{i}^{*}C_{j} + C_{j}C_{i}^{*}) \cos(\eta_{B} - \eta_{B'}) \tag{4.43}\]

does not violate the invariance under \(P\), nor under \(C\), nor under \(T\); while the presence of a term in the form

\[(O_{+})_{\text{scalar}} = \sum iO_{iB_{j}B'}(C_{i}^{*}C_{j} - C_{j}^{*}C_{i}) \sin(\eta_{B} - \eta_{B'}) \tag{4.44}\]

violates the invariance under \(C\) and \(T\), but not under \(P\) [e.g., the \((p_{e} \cdot p_{p})Z\) term in Equation (5.3)].

(ii) Let \(O_{-}\) be a pseudoscalar but still be even in \(p\) and \(s\) (e.g., \(s \cdot p\)). The observation of such a term in the form
\[ (O_{\ast})_{ps} = \sum O_{iB,jB'} (C_i^* C_j + C_i C_j^*) \cos(\eta_B - \eta_{B'}) \]  

(4.45)

violates the invariance under \( P \) and \( C \), but not under \( T \) [e.g., the \( Z \) independent term of \( s \cdot p_z \) in Equation (5.11)]; while the presence of a term in the form

\[ (O_{\ast})_{ps} = \sum iO_{iB,jB'} (C_i^* C_j C_i C_j^*) \sin(\eta_B - \eta_{B'}) \]  

(4.46)

violates the invariance under \( P \) and \( T \), but not under \( C \) [e.g., the \((s \cdot p_z)Z\) term in Equation (5.11)].

(iii) Let \( O_\ast \) be a scalar and be odd in \( p \) and \( s \) [e.g., \( s \cdot (p_1 \times p_2) \)]. The observation of such a term in the form

\[ (O_{\ast})_{\text{scalar}} = \sum O_{iB,jB'} (C_i^* C_j C_i C_j^*) \cos(\eta_B - \eta_{B'}) \]  

(4.47)

violates the invariance under \( T \) and \( C \), but not under \( P \); while the existence of a term in the form

\[ (O_{\ast})_{\text{scalar}} = \sum iO_{iB,jB'} (C_i^* C_j C_i C_j^*) \sin(\eta_B - \eta_{B'}) \]  

(4.48)

does not violate the invariance under \( P \), not under \( C \), nor under \( T \). A particular example of (4.48) is the existence of the usual production of polarized nuclei by a single scattering. Because of time reversal invariance it follows that such polarization is zero in the Born approximation.

(iv) Let \( O_\ast \) be a pseudoscalar and be odd in \( p \) and \( s \) [e.g., \( p_1 \cdot (p_2 \times p_3) \)]. The observation of such a term in the form

\[ (O_{\ast})_{ps} = \sum O_{iB,jB'} (C_i^* C_j C_i C_j^*) \cos(\eta_B - \eta_{B'}) \]  

(4.49)

violates the invariance under \( P \) and \( T \), but not under \( C \); while the existence of a term like

\[ (O_{\ast})_{ps} = \sum iO_{iB,jB'} (C_i^* C_j C_i C_j^*) \sin(\eta_B - \eta_{B'}) \]  

(4.50)

violates the invariance under \( P \) and \( C \), but not under \( T \).
V. VARIOUS POSSIBLE EXPERIMENTAL TESTS\textsuperscript{25} ON INVARiance UNDER P, C, AND T IN WEAK INTERACTIONS

1. Electron-Neutrino Correlation in $\beta$ Decay

We take for the $\beta$ decay Hamiltonian

$$H = \sum (\psi^+_i O_i \psi_i) [C_i (\psi^+_i O_i \psi_i) + C_i (\psi^+_i O_i \psi_i)]$$  \hspace{1cm} (5.1)

where $i$ runs over the usual $S$, $V$, $T$, $A$, and $P$ types of interactions with

$$O_S = \gamma_4, \quad O_V = \gamma_\mu \gamma_5, \quad O_T = -[i/(2\sqrt{2})] \gamma_4 (\gamma_\mu \gamma_5 - \gamma_\mu \gamma_5),$$

$$O_A = -i \gamma_4 \gamma_5 \gamma_5, \quad \text{and} \quad O_P = \gamma_4 \gamma_5.$$  \hspace{1cm} (5.2)

The angular energy distribution of the electron for the allowed transition is given by\textsuperscript{26}

$$N(W, \theta) dW = \frac{\xi}{4\pi^3} F(Z, W') p W(W_0 - W)^2$$

$$\times \left[ 1 + \frac{p}{W} \cos \theta (a + a' \frac{Ze^2}{\hbar c p}) + \frac{b}{W} \right] \sin \theta d\theta$$  \hspace{1cm} (5.3)

where

$$\xi = (C_\delta)^2 + (C_V)^2 + (C_\alpha)^2 + (C_\nu)^2 |M_\varepsilon|^2 +$$

$$(|C_T|^2 + |C_4|^2 + |C''|^2 + |C_V|^2) |M_{\sigma T}|^2,$$  \hspace{1cm} (5.4)

$$a\xi = (\gamma_5) (|C_T|^2 - |C_\delta|^2 + |C''|^2 - |C_V|^2 |M_{\sigma T}|^2 -$$

$$(|C_\alpha|^2 - |C_\nu|^2 + |C_\nu'|^2 - |C_V'|^2) |M_\varepsilon|^2,$$  \hspace{1cm} (5.5)

$$a'\xi = (i/3) (C_\delta C_T' - C_T C_\delta' + C_4' C_V' - C_\nu' C_V'' |M_{\sigma T}|^2 +$$

$$i (C_\delta C_\nu - C_\delta C'_\nu + C_\nu'' C''_4 - C''_\nu C_\nu'') |M_\varepsilon|^2,$$  \hspace{1cm} (5.6)

$$b\xi = \gamma (C_\delta C_V + C_\delta C_\nu + C_\nu'' C'_\nu + C_\nu' C''_V) |M_\varepsilon|^2 +$$

$$\gamma (C_T C_\delta + C_T C_\nu + C_V'' C_\nu' + C_\nu' C''_V) |M_{\sigma T}|^2.$$  \hspace{1cm} (5.7)

\textsuperscript{25}Since January 1957 a large number of experiments have been performed to test the non-conservation properties of $P$ and $C$ in weak decays. For a review of these numerous experimental works, see, e.g., *Proc. Seventh Ann. Rochester Conf.*, Interscience, New York, 1957.

\textsuperscript{26}Cf. Equation (A.4) of reference 14. The existence of the term $a' Ze^2/\hbar c p$ was pointed out by M. Morita and R. Curtis.
In the above expressions, all unexplained notations are identical with the standard notations.\textsuperscript{27} Equations (5.4) to (5.8) can be obtained directly by applying the general rules, Equation (3.21), to the corresponding "old" expressions calculated previously under the assumption that parity is conserved.

We notice that the term containing

\[ a'Z(p_e \cdot p_e) \]  \hspace{1cm} (5.9)

in (5.3) is of the form described by Equation (4.44) with the phase shifts due to the Coulomb effect ($p_e, p_e$ are respectively the momenta of $e$ and $\nu$). Thus, the presence of this term would violate the invariance under $C$ and $\mathcal{T}$.

2. Experiments with $\beta$ Decay from Polarized Nuclei\textsuperscript{14}

We consider first the experiment on angular distribution for allowed $\beta$ transition from polarized nuclei. Let $\theta$ be the angle between $p_e$ and direction of the spin $J$ of the polarized nuclei. The angular distribution is, in general, of the form

\[ 1 + \alpha \cos \theta. \]  \hspace{1cm} (5.10)

The corresponding expression $\alpha$ for $J \rightarrow J - 1$ (no) transition is

\[ \alpha = \beta \frac{\langle J_z \rangle}{J}. \]

with

\[ \beta = \text{Re} \left[ \pm \left( C_T C_T^* - C_A C_A^* \right) + i \frac{Ze^2}{\hbar c} \left( C_A C_T^* + C_T C_A^* \right) \right] \frac{v_e}{\xi} \frac{2}{\xi + \left( \xi b / W \right)} |M_{\sigma r}|^2. \]  \hspace{1cm} (5.11)

For $J \rightarrow J + 1$ (no), $\alpha$ is given by

\[ \alpha = - \beta J_z / (J+1). \]  \hspace{1cm} (5.12)

For $J \rightarrow J$ (no), the corresponding $\alpha$ is\textsuperscript{28}

\[ \alpha = \beta \frac{J_z}{J(J+1)} + \beta' \frac{J_z}{\sqrt{J(J+1)}}. \]  \hspace{1cm} (5.13)

where

\[ \beta' = \text{Re} \left[ \left( C_A C_A^* + C_B C_B^* - C_A C_B^* - C_B C_A^* \right) \right] \frac{i Ze^2}{\hbar c} \left( C_A C_A^* + C_B C_B^* - C_A C_B^* - C_B C_A^* \right) \frac{v_e}{\xi} \frac{2M_p M_{\sigma r}}{\xi + \left( \xi b / W \right)}. \]

\textsuperscript{27}See, e.g., the article by M.E. Rose in Beta- and Gamma-Ray Spectroscopy, Interscience, New York, 1955, pp. 271-91.

\textsuperscript{28}Equation (5.13) has been independently calculated by many authors: M. Morita (private communication); R. Curtis and R. Lewis (private communication); J.D. Jackson, S.B. Treiman, and H.W. Wyld (Phys. Rev., in press). In deriving Equation (5.13) we assume that the strong interactions are invariant under time reversal. Consequently the nuclear matrix elements $M_F$ and $M_{\sigma r}$ are real quantities. If the strong interactions are not invariant under time reversal then the product $(M_F M_{\sigma r})$ should be replaced by $(M_F^* M_{\sigma r})$ and set inside the square brackets together with the coupling constants. It may be emphasized that the condition $\beta' = 0$ can be used as evidence of invariance under time reversal for both the weak and the strong interactions.
In Equations (5.11) to (5.13) the upper signs are for $\varepsilon^-$ emission and the lower signs for $\varepsilon^+$ emission.

The detection of $\alpha \neq 0$ gives definite proof that $P$ is not conserved. We notice further that in the expression for $\beta$ [Equation (5.11)] there are two different terms of the forms of $J \cdot p_e$ and $ZJ \cdot p_e$. Comparing with Equations (4.45) and (4.46) we see that the presence of the first term violates the invariance of $P$ and $C$ while the presence of the second term violates the invariance of $P$ and $T$.

The first conclusive experimental evidence on the nonconservation of parity in weak interactions was done by Wu, Ambler, Hayward, Hoppes, and Hudson\textsuperscript{1} using polarized Co\textsuperscript{60}. The decay scheme for Co\textsuperscript{60} is

$$\text{Co}^{60} \rightarrow \text{Ni}^{60} + \varepsilon^- + \bar{\nu}$$  \hspace{1cm} (5.14)

with $J = 5 \rightarrow J = 4$ (no) and $\nu_e / c \approx 0.65$. Wu et al. obtained for the asymmetry parameter $\beta$,

$$\beta \approx -0.7.$$  \hspace{1cm} (5.15)

Since from the He\textsuperscript{6} recoils we know that

$$\frac{|C_4|^2 + |C_4'|^2}{|C_\tau|^2 + |C_\tau'|^2} < \frac{1}{3}$$  \hspace{1cm} (5.16)

and since for Ni\textsuperscript{60}

$$Ze^2/\hbar c = 0.2,$$

the second term in the expression for $\beta$ [Equation (5.11)] has an upper limit

$$\left| \frac{2(\nu_e / c) Re[i(Ze^2/\hbar c)(C_4 \cdot C_4' + C_4' \cdot C_\tau')]}{|C_4|^2 + |C_4'|^2 + |C_\tau|^2 + |C_\tau'|^2} \right| < 0.23.$$  \hspace{1cm} (5.17)

Thus from the observed magnitude [Equation (5.15)] we conclude that in $\beta$ decay both $C$ and $P$ are not conserved. By more accurate measurement to study the $Z$ dependence or $p$ dependence of the asymmetry parameter $\beta$ and $\beta'$ one can obtain information concerning time reversal invariance.

3. Other $\beta$ Decay Experiments

There are other experiments in $\beta$ decay which can serve as tests for possible non-invariance under $P$, $C$, and $T$. We list the following:

A. $\beta$-$\gamma$ CORRELATION AND THE CIRCULAR POLARIZATION OF THE $\gamma$-RAY\textsuperscript{11}

Consider a successive $\beta, \gamma$ decay scheme

$$A \rightarrow B^* + \varepsilon^- + \nu \quad \text{and} \quad B^* \rightarrow B + \gamma$$  \hspace{1cm} (5.18)

in which the initial nucleus $A$ is not polarized (see Figure 1). Because of the nonconservation of parity, the intermediate nucleus $B^*$ will be polarized along the momentum $p_e$ of the $\beta$-ray. The polarization of the $B^*$ can then be detected by measuring the direction of the $\gamma$-ray with respect to $p_e$ and the state of circular polarization of the $\gamma$-ray. The magnitude of the polarization of $B^*$ can be easily deduced from Equation (5.10).

B. POLARIZATION OF $e^-$

By measuring the longitudinal polarization of the electron $(\sigma \cdot p_e)$ from unpolarized nuclei one can also obtain a test of parity conservation.\textsuperscript{30} By a detailed study of its possible Z dependence it is also possible to obtain information on time reversal invariance. (See Chapter VI, section 2, for a more detailed discussion.)

C. POLARIZED NUCLEI AND RECOIL EXPERIMENTS\textsuperscript{31}

From the experiments of Wu et al., it is proven that parity and charge conjugation are not conserved in $\beta$ decay. The next important question is, of course, on the invariance under time reversal $T$. (By CPT theorem this is equivalent to the invariance under $C \cdot P$.) In the above mentioned experiments, by a careful study of $Z$ dependent terms such as

$$ (J \cdot p_e)Z \quad \text{and} \quad (p_e \cdot p_e)Z, \quad (5.19) $$

information on $T$ invariance can be obtained. While their presence would prove that we do not have invariance under time reversal, their absence may be due to other reasons. For example, in Equations (5.7) and (5.11), if

$$ C_A = C_A' = 0 \quad \text{and} \quad C_V = C_V' = 0 $$


then these terms [Equation (5.19)] would also vanish. A critical test is then to measure possible time reversal nonconservation terms that are due to the interference terms between \( C_8 \) and \( C_T \). One such possibility has been pointed out by Jackson, Trieman, and Wyld.\(^{31}\)

Let us consider, for example, the neutron decay from polarized neutrons

\[
n \rightarrow p + e^- + \bar{\nu}.
\]  

(5.20)

A simultaneous measurement of \( p_e \) and the recoil proton can give a measurement of

\[
\sigma_n \cdot (p_e \times p_p).
\]

(5.21)

From the previous general argument, Equation (4.47), we see that the \( Z \) independent term of this quantity violates the invariance under \( T \) and \( C \). The coefficient of this term is proportional to

\[
\text{Im}(C_8 C_T^* - C_Y C_A^* + C_S' C_T^* - C_Y' C_A'^*)
\]

(5.22)

4. \( \pi \) Decay\(^{14,23}\)

Consider the reaction for \( \pi^- \) decay

\[
\pi^- \rightarrow \mu^- + \nu.
\]

(5.23)

The parity nonconservation for this reaction can be established by measuring the longitudinal polarization of the \( \mu \)-meson. If the \( \mu \)-meson has spin \( \frac{1}{2} \), its polarization state is described by a density matrix

\[
1 + A \cdot \sigma_\mu \cdot \hat{p}_\mu
\]

(5.24)

where \( \hat{p}_\mu \) is a unit vector along the momentum of \( \mu \). Assuming that \( \mu \) and \( \nu \) have no strong interactions between them, from Equation (4.45) we see that the presence of \( A \), violates the conservation laws of \( P \) and \( C \).

The density matrix for the polarization of the \( \mu^- \) from the corresponding \( \pi^- \) decay,

\[
\pi \rightarrow \mu^- + \nu
\]

(5.25)

is

\[
1 + A \cdot \sigma_\mu \cdot \hat{p}_\mu.
\]

(5.26)

By \( CPT \) theorem, the final result must be invariant under the operation of \( C \cdot P \cdot T \). Since there are no final state interactions, we see that

\[
A_+ = -A_-
\]

(5.27)

5. \( \mu \) Decay\(^{14,23}\)

The easiest way to analyze the spin state of the \( \mu^- \)-meson from \( \pi \) decay is to use the possible parity nonconservation terms in \( \mu \) decay

\[
\mu^- \rightarrow e^- + \nu + \bar{\nu}.
\]

(5.28)
If the $\mu$-meson is polarized with spin $\sigma_\mu$, the angular distribution of $e^-$ would be of the form

$$1 + B_\mu \sigma_\mu \cdot \hat{p}_e$$  \hspace{1cm} (5.29)

where $\hat{p}_e$ is a unit vector along the momentum of $e$. Similar to (5.27), we have \footnote{If in the $\mu^-$ decay two neutrinos are emitted, $\mu^- \rightarrow e^- + 2\nu$, then the corresponding $\mu^+$ decay is $\mu^+ \rightarrow e^+ + 2\bar{\nu}$. In this case Equation (5.30) is still correct.}

$$B_\mu = -B_\pi.$$  \hspace{1cm} (5.30)

Combining (5.24), (5.26), and (5.29), in the decays

$$\pi^+ \rightarrow \mu^+ \rightarrow e^\pm$$

the angular correlation between $p_\pi$ and $p_\mu$ is of the form

$$1 + \alpha \cdot \hat{p}_\mu \cdot \hat{p}_e$$  \hspace{1cm} (5.31)

with $p_\pi$ measured in the rest system of $\pi^+$ and $p_\mu$ in the rest system of $\mu^+$. Furthermore, from (5.27) and (5.30), we have

$$\alpha_\pi = -\alpha_\mu.$$  \hspace{1cm} (5.32)

The observation of $\alpha_\pi$ proves that $P$ and $C$ are not conserved in both $\pi$ decay and $\mu$ decay.

Experiments\footnote{More recently T. Coffin et al. \textit{[Phys. Rev.} 106, 1108 (1957)] measured the $g$ value for $\mu^-$ using the magnetic resonance technique. They found $g_\mu = +2.0064 \pm 0.0048$.} on $\pi$-$\mu$-$e$ decays give for $\alpha_\pi$,

$$\alpha_\pi = -0.26 \pm 0.02$$  \hspace{1cm} (5.33)

with $\mu^+$ stopped in carbon and

$$\alpha_\mu = -0.16$$  \hspace{1cm} (5.34)

with $\mu^+$ stopped in emulsion. The difference between these two numbers is, of course, due to the large depolarization effect in emulsion.

The longitudinal polarization of the $\mu$-meson from $\pi$ decay offers a natural possibility for measuring the magnetic moment of the $\mu$-meson. This was first measured by Garwin et al.\footnote{More recently T. Coffin et al. \textit{[Phys. Rev.} 106, 1108 (1957)] measured the $g$ value for $\mu^-$ using the magnetic resonance technique. They found $g_\mu = +2.0064 \pm 0.0048$.} Their result gives a $g$ value\footnote{More recently T. Coffin et al. \textit{[Phys. Rev.} 106, 1108 (1957)] measured the $g$ value for $\mu^-$ using the magnetic resonance technique. They found $g_\mu = +2.0064 \pm 0.0048$.} for $\mu^+$,

$$g = +2.00 \pm 0.10.$$  \hspace{1cm} (5.35)

This value strongly indicates that the spin of $\mu$ is $\frac{1}{2}$. A more accurate measurement of this is of particular interest because it may give a very severe test of validity electrodynamics at a much smaller distance ($\sim 10^{-13}$ cm) than that tested by previous experiments with electrons.

6. $K^+$ Decay

We consider first the decay of $K^+_{\mu2}$

$$K^+ \rightarrow \mu^+ + \nu \ (or \ \bar{\nu})$$  \hspace{1cm} (5.36)

As in the $\pi$-$\mu$-$e$ decay, one would expect that here again the $\mu$-meson could be polarized along its direction of motion, and then the $\mu$-$e$ decay would be an analyzer
of the polarization through measurements of the distribution of \( p_\mu \cdot p_e \). We will show later, in a special two-component theory of the neutrino, that this electron distribution is identical with the distribution from the \( \pi^\pm \mu^-e^- \) decay if the \( K^- \) meson spin is zero. Therefore in this special theory the \( K^-\mu^-e^- \) decay distribution may offer a way to obtain some information about the spin of the \( K^- \) meson\(^{23} \) (cf. Chapter VII, section 1).

Next we consider the decays of \( K_{\pi^2} \) and \( K_{\pi^3} \):

\[
K^+ \rightarrow \pi^+ + \pi^0 \tag{5.37}
\]

and

\[
K^+ \rightarrow \pi^- + 2\pi^+ . \tag{5.38}
\]

In the decay of \( K_{\pi^2} \) only one independent momentum can be measured in the rest system of \( K \), and in the decay of \( K_{\pi^3} \) only two independent momenta can be measured. In neither case can one form a pseudoscalar quantity out of these observed momenta. Thus, if the \( K^- \) meson has zero spin (or if it is unpolarized) it is impossible to observe any interference term between the even and odd states in the decay of \( K_{\pi^2} \) and \( K_{\pi^3} \). The strongest evidence for nonconservation of parity in this case is precisely the present \( \theta^-\tau \) problem, namely, the \( K^- \) meson can decay into a \( 2\pi \) system and a \( 3\pi \) system with the same lifetime and same mass.

The determination of the spin-parity of the \( K^- \) meson through the angular energy distribution of \( 3\pi \) mode (Dalitz’s analysis) can still be used. The distribution function is now given by

\[
[|\psi_{J^+}(k,p)|^2 + \alpha|\psi_{J^+}(k,p)|^2] \sqrt{\epsilon(1-\epsilon)} \, d\epsilon \, d(\cos\theta) \tag{5.39}
\]

where \( \psi_{J^+} \) and \( \psi_{J^+} \) are the wave functions for a \( 3\pi \) system with total angular momentum \( J \) and parity \(+1\) and \(-1\) respectively [cf. Equation (2.6)]. The constant \( \alpha \) is a real positive number. It is to be expected that \( J=0 \) or 2 would still be favorable.

7. \( \Lambda^0 \) Decay and \( \Sigma \) Decay

Information concerning parity nonconservation can be obtained by studying the decay of hyperons.\(^{14} \) We consider as an example the following reactions:

\[
\pi^- + p \rightarrow \Sigma^+ + K^+ \tag{5.40}
\]

and

\[
\Sigma^- \rightarrow n + \pi^- . \tag{5.41}
\]

The first reaction (5.40) can be thought of as the polarizer of \( \Sigma^- \). Since it is a strong interaction, in order to produce a polarized \( \Sigma^- \) it is necessary to measure the direction of two momenta, say \( p_1 \) and \( p_2 \), the momentum of the incoming pion, and \( p_2 \), the momentum of \( \Sigma^- \). The spin \( s_\Sigma \) of the \( \Sigma^- \) then will be polarized along the direction \( (p_1 \times p_2) \). In the subsequent decay (5.41), if in the angular distribution of \( p_{\text{out}} \) (the momentum of the decay pion in the rest system of \( \Sigma^- \)) a term of the form \( s_\Sigma \cdot p_{\text{out}} \) is observed, then parity is not conserved. Because what one measures is a quantity of the form \( p_{\text{out}} \cdot (p_1 \times p_2) \), it is necessary to exclude cases where \( p_1 \) is parallel to \( p_2 \).
We shall give a more detailed consideration of processes (5.40) and (5.41) by further assuming that (i) the spin of $\Sigma$ is $\frac{1}{2}$, (ii) the spin of $K$ is zero, and (iii) in the production process only $s$ and $p$ waves are important. Under these assumptions, the differential production cross section per unit solid angle $d\Omega$ (in the center-of-mass system of production) of the $\Sigma^-$ produced is given by

$$I(\theta) = |a + b \cos \theta|^2 + |c|^2 \sin^2 \theta$$

(5.42)

where $\theta = \angle (p_{1\,\Sigma}, p_\Sigma)$.

The corresponding polarization $P(\theta)$ is

$$P(\theta) = [I(\theta)]^{-1/2} \sin \theta \cdot \text{Im}[c^*(a + b \cos \theta)]$$

(5.43)

where $P(\theta)$ is defined to be the average spin of the $\Sigma^-$ in units of $(\frac{1}{2})\hbar$. It is important to notice that if $I(\theta) = (1 \pm \cos \theta)^2$ then $P(\theta) = 0$ at all angles.

Because of the possible nonconservation of parity in the decay process (5.41) the polarization $P(\theta)$ can be measured. Let $R$ be the projection of $p_{\text{out}}$ in the direction of $p_{1\,\Sigma} \times p_\Sigma$. The distribution function for $R$ at an angle $\theta$ of production is given by

$$W(\theta, \xi)d\Omega d\xi = I(\theta) d\Omega \cdot (\frac{1}{2}) [1 + \alpha P(\theta) \xi] \alpha \xi$$

(5.44)

where

$$\xi = R/(\text{maximum value of } R) \equiv R/(100 \text{ Mev/c})$$

In terms of the coefficients $a$, $b$, and $c$, defined in (5.42), $W(\theta, \xi)$ can be written as

$$W(\theta, \xi)d\xi d\Omega = (|a + b \cos \theta|^2 + |c|^2 \sin^2 \theta)(\frac{1}{2})d\xi d\Omega + \alpha \sin \theta \text{Im}[c^*(a + b \cos \theta)] \xi \xi d\Omega$$

(5.45)

The existence of a nonvanishing $\alpha$ constitutes an unambiguous proof of parity nonconservation in $\Sigma^-$ decay. In such a case the final state of $(n + \pi^-)$ in process (5.41) is a mixture of $s_{1/2}$ and $p_{1/2}$ states with amplitudes, say, $A$ and $B$ respectively. The asymmetry parameter $\alpha$ is related to these amplitudes by

$$\alpha = \frac{2Re(A^*B)}{|A|^2 + |B|^2}$$

(5.46)

If time reversal leaves invariant the decay process of $\Sigma^-$ then (cf. Chapter IV, section 2)

$$\alpha = \pm \frac{2|A|\cdot |B|}{|A|^2 + |B|^2} \cos(\delta_p - \delta_s)$$

(5.47)

where $\delta_p$ and $\delta_s$ are, respectively, the phase shifts of $(n + \pi^-)$ scattering in the $p_{1/2}$ and $s_{1/2}$ states at about 117 Mev in their center-of-mass system. If the decay interaction is invariant under charge conjugation then (cf. Chapter IV, section 3)

$$\alpha = \pm \frac{2|A|\cdot |B|}{|A|^2 + |B|^2} \sin(\delta_p - \delta_s)$$

(5.48)

---

Thus a large observed value of $\alpha$ means that both $C$ and $P$ are not conserved in the decay of $\Sigma^-$. Similar considerations can easily be applied to the productions and decays of other hyperons.

8. $\Xi$ Decay

Another example of a possible test of parity nonconservation is through the decay of the $\Xi$-particle.\textsuperscript{14}

\[ \Xi^- \rightarrow \Lambda^0 + \pi^- \]  
(5.49)

and

\[ \Lambda^0 \rightarrow p + \pi^- . \]  
(5.50)

As in the case of $\pi$-$\mu$-$\epsilon$ decay, if parity is not conserved in both (5.49) and (5.50), then the distribution of the momentum of the $\Lambda^0$, $p_{\Lambda}$, and that of the pion, $p_{\pi}$, may contain odd powers of

\[ (p_{\Lambda} \cdot p_{\pi}) \]  
(5.51)

where $p_{\pi}$ is measured in the rest system of $\Lambda^0$ and $p_{\Lambda}$ in the rest system of $\Xi^-$. We shall illustrate the calculation of such a distribution by considering the special case that the spins of $\Lambda^0$ and $\Xi^-$ are both $\frac{1}{2}$. Let $\phi_{\Xi}$ be the initial spin state of $\Xi^-$ at rest. The wave function of the corresponding $\Lambda^0$ in the decay (5.49) can be written as

\[ \phi_{\Lambda} = M_{\Lambda} \cdot \phi_{\Xi} \]  
(5.52)

where

\[ M_{\Lambda} = A + B \sigma \cdot \hat{p}_{\Lambda} \]

with $\hat{p}_{\Lambda}$ a unit vector along $p_{\Lambda}$ and $A$, $B$ the relative probability amplitudes of the two final states of opposite parity ($|A|^2 + |B|^2 = 1$). The subsequent wave function of $\rho$ in (5.50) is

\[ \phi_{\rho} = M_{\rho} \cdot \phi_{\Lambda} \]  
(5.53)

where

\[ M_{\rho} = a + b \sigma \cdot \hat{p}_{\pi}, \quad (|a|^2 + |b|^2 = 1) \]

with $\hat{p}_{\pi}$ a unit vector along $p_{\pi}$.

The final distribution of $p_{\pi}$ and $p_{\Lambda}$ for an unpolarized $\Xi$ is

\[ \frac{1}{2} \sum |\phi_{\rho}|^2 = \left( \frac{1}{2} \right) \text{tr.} [M_{\Lambda}^* M_{\rho}^* M_{\rho} M_{\Lambda}] = 1 + \alpha \cos \theta \]  
(5.54)

where

\[ \theta = \angle (p_{\Lambda}, p_{\pi}) \quad \text{and} \quad \alpha = (a^* b + b^* a) (A^* B + B^* A). \]

In Equation (5.54), the sum extends over the two initial spin states of $\phi_{\Xi} (= \uparrow, \downarrow)$. Thus the observation of a nonvanishing value for $\alpha$ shows that $P$ is not conserved in both $\Xi^-$ decay and $\Lambda^0$ decay.

9. $K^0$, $\bar{K}^0$ Decay\textsuperscript{22}

As a final example we consider the decay of neutral $K$-mesons. The existence of two neutral $K$-particles of different lifetimes and many of their properties were pre-
dicted and discussed under the assumption that the decay of the $K$-meson is strictly invariant under charge conjugation $C$. Although there is as yet no explicit proof that $C$ is not conserved in the decay of the $K$-particle, the various recent experiments on parity and charge conjugation nonconservations in other weak interactions do give a strong indication that $C$ is probably not invariant in $K$ decay. As we shall see, the curious behavior of $K^0$ and $\bar{K}^0$ turns out to be remarkably insensitive to any possible nonconservation of $P$, $C$, or $T$. From the strong production processes we know that there must exist two different states, $K^0$ and $\bar{K}^0$, of opposite strangeness quantum number [Equation (1.27)]. Thus, independent of any assumption about the invariance or non-invariance properties, there should exist, in general, two lifetimes for their decay. We shall discuss the $K-\bar{K}$ decay in some detail under the following two possibilities.

A. THE DECAY PROCESSES ARE INVARIANT UNDER $T$

First we consider the various consequences if under time reversal $T$ the decay processes are invariant. It follows, then, from the CPT theorem that $C-P$ is conserved, although $C$ may not be conserved. We define the states $|K_1\rangle$ and $|K_2\rangle$ by

$$|K_1\rangle = \frac{1}{\sqrt{2}} \left( |K^0\rangle + C-P |K^0\rangle \right) \quad \text{and} \quad |K_2\rangle = \frac{1}{\sqrt{2}} \left( |K^0\rangle - C-P |K^0\rangle \right).$$

(5.55)

Thus, $|K_1\rangle$ and $|K_2\rangle$ are eigenstates of $C-P$ with eigenvalues $+1$ and $-1$ respectively. If $C-P$ is strictly invariant, $|K_1\rangle$ and $|K_2\rangle$ each will decay exponentially in time. The lifetime of $|K_1\rangle$ is, in general, different from that of $|K_2\rangle$. Furthermore

$$K_1^0 \rightarrow \pi^+ + \pi^-$$

(5.56)
in addition to its other possible modes of decay. [For example, if $\langle \text{spin}\rangle_{K^0} = 0, 2, 4 \ldots$, then $K_1^0$ also $\rightarrow 2\pi^0$.] On the other hand $K_2^0$ cannot decay into a $2\pi$ system. This follows from the fact that a $2\pi$ system with total spin $J$ is always an eigenstate of $C-P$ with eigenvalue $+1$. From phase space arguments, it is expected that $K_1^0$ has a shorter lifetime.

In order to detect any difference between the present case and the case when $C$ is conserved it is necessary to measure a pseudoscalar quantity. For example, in the decay of the long-lived $K$-particle, $K_2^0$,

$$K_2^0 \rightarrow \pi^+ + e^- + \bar{\nu},$$

(5.57)
the electron may be longitudinally polarized and its polarization $(\sigma \cdot p)_\epsilon$ may be measured. The corresponding quantity $(\sigma \cdot p)_\nu$ for the positron in the decay,

$$K_2^0 \rightarrow \pi^- + e^+ + \nu,$$

(5.58)
is expected to be of opposite sign from that of $(\sigma \cdot p)_\epsilon$. However in a measurement which does not involve a pseudoscalar quantity, the nonconservation of $C$ can not be tested. For example, let $r$ be the branching ratio

\[ M. \text{Gell-Mann and A. Pais, Phys. Rev. 97, 1387 (1955).} \]
\[ r = \frac{\text{rate of } K^0 \rightarrow \pi^+ + e^- + \bar{\nu}}{\text{rate of } \bar{K}^0 \rightarrow \pi^- + e^+ + \nu}. \quad (5.59) \]

We should expect \( r = 1 \) in the present case.

B. \( K^0 \) AND \( \bar{K}^0 \) DECAY PROCESSES MAY NOT BE INVARIANT UNDER \( T \)

Next we consider the more general case that under time reversal \( T \) the decay processes may not be invariant. We define \( |\bar{K}^0\rangle \) to be

\[ |\bar{K}^0\rangle = C \cdot |K^0\rangle \quad (5.60) \]

where the charge conjugation operator \( C \) is defined through the strong interaction. Suppose at \( t = 0 \) a \( K \)-particle is produced. At a later time its wave function can be described as

\[ \psi(t) = a(t)|K^0\rangle + b(t)|\bar{K}^0\rangle, \quad (5.61) \]

or simply as

\[ \psi(t) = \begin{pmatrix} a(t) \\ b(t) \end{pmatrix}. \quad (5.62) \]

The differential equation for \( \psi(t) \) can be written as

\[ -i\hbar \frac{d\psi}{dt} = (\Gamma + iM)\psi, \quad (5.63) \]

where

\[ \Gamma_{11} = \Gamma_{22} = \sum_j \Gamma_{aj} = \sum_j \Gamma_{bj}, \quad \Gamma_{12} = \Gamma_{21}^* = \sum_j (\Gamma_{aj} \bar{\Gamma}_{bj})^{1/2} e^{i\delta_{aj}}, \quad (5.64) \]

with

\[ \Gamma_{aj} = 2\pi|H_{aj}|^2 \times \text{(density of states per unit energy)}; \]

\[ \Gamma_{bj} = 2\pi|H_{bj}|^2 \times \text{(density of states per unit energy)}; \]

\[ e^{i\delta_{aj}} = \text{phase of } (H_{aj}/H_{bj}); \]

where \( H_{aj} \) and \( H_{bj} \) are the matrix elements of the Hamiltonian for the decay processes, \( a \) and \( b \) refer to states \( |K^0\rangle \) and \( |\bar{K}^0\rangle \) respectively, and \( j \) refers to any possible decay state (consisting of standing waves) that is an eigenstate of \( H_{\text{strong}} \). Thus state \( |j\rangle \) has a definite spin, charge, and parity. That \( \Gamma_{11} = \Gamma_{22} \) follows from

\[ H_{aj}^* = \pm H_{bj}, \quad (5.65) \]

with \( |\tilde{j}\rangle = C|j\rangle \). Equation (5.65) is an immediate consequence of the CPT theorem [cf. Equation (4.14)].

Similarly, we have for the mass operator \( M \),

\[ M^\dagger = M \quad \text{and} \quad M_{11} = M_{22}. \quad (5.66) \]

Equation (5.63) can now be readily solved. Its eigenstates, defined by

\[ (\Gamma + iM)\psi = \lambda\psi, \]

are

\[ \psi = \begin{pmatrix} p \\ \pm q \end{pmatrix} (|p|^2 + |q|^2)^{-1/2}, \quad (5.67) \]

with the corresponding time constants
\[ \lambda = \Gamma + i M, \pm (pq), \]  

(5.68)

where \( p \) and \( q \) are two complex numbers given by

\[ p^2 = \Gamma + i M \quad \text{and} \quad q^2 = \Gamma + i M. \]  

(5.69)

If at \( t = 0 \) a \( K^0 \)-particle is produced, then at a later time the state function \( \psi \) can be expressed in terms of these two eigenstates \( \psi_+ \) as

\[ \psi(t) = \frac{1}{\sqrt{2}} (|p|^2 + |q|^2)^{1/2} (\psi^\lambda e^{\lambda t/2} + \psi^\lambda e^{-\lambda t/2}). \]  

(5.70)

It is convenient to separate the real and imaginary parts of \( \lambda \). Without loss of generality we may write

\[ \lambda = \gamma + 2i \Delta, \]  

(5.71)

where \( \gamma, \gamma \) are two real numbers representing the reciprocal lifetimes of the short-lived ones and the long-lived ones respectively, and \( \Delta \) is the mass difference between these two eigenstates. One notices that these two eigenstates \( \psi_+ \) and \( \psi_- \) do not in general represent the states \( K_1 \) and \( K_2 \) introduced in (5.55). In fact they may not be orthogonal to each other.

The fractional number of \( K \)-mesons that decay at time \( t \) after production is given by

\[ N(t) dt = -d(\psi\psi^\dagger). \]  

(5.72)

Using (5.63) one easily shows that

\[ -\frac{d}{dt}(\psi\psi^\dagger) = \psi^\dagger \Gamma \psi. \]

By using Equations (5.67) to (5.70), Equation (5.72) becomes

\[ N(t) = (\frac{1}{2}) (1 + \alpha) \left\{ \gamma e^{-\gamma t} + \gamma e^{\gamma t} + \alpha e^{-(\gamma + \gamma) t/2} [(\gamma + \gamma) \cos \Delta t - 2 \Delta \sin \Delta t] \right\}, \]  

(5.73)

where

\[ \alpha = \frac{\psi^\dagger \psi}{(|p|^2 - |q|^2)(|p|^2 + |q|^2)^{-1}} \]  

(5.74)

is a real number representing the non-orthogonality of these two eigenstates. The four real numbers \( \gamma, \gamma, \Delta, \) and \( \alpha \) characterize the decay of the \( K \)-particle. They satisfy the inequalities

\[ \gamma \geq 0 \quad \text{and} \quad \alpha^2 \leq \frac{4\gamma \gamma}{(\gamma + \gamma)^2 + 4\Delta^2} \]  

(5.75)

which follow from the fact that \( \Gamma \) is a positive Hermitian matrix. These conditions also insure that \( N(t) \geq 0 \) for all \( t \).

Experimentally \( N(t) \) is measurable. From \( N(t) \) one can in principle determine all four constants \( \gamma, \gamma, \Delta, \) and \( \alpha \). Indications from existing experiments\(^{36}\) show that probably \( (\gamma \gamma) \geq 100 \). Equation (5.75) then shows that \( \alpha^2 < 4(\gamma \gamma) < 0.04 \).

The above discussion also leads easily to a determination of the branching ratio of the long-lived component (and the short-lived component) into the various decay

modes. If time reversal is invariant the long-lived component is an eigenstate of C-P. As discussed in part A above, its decay into charge conjugate channels such as $\pi^+ + e^- + \bar{\nu}$ and $\pi^- + e^- + \nu$ must be equally probable. If time reversal is not strictly invariant, decays into $\pi^+ + e^- + \bar{\nu}$ and $\pi^- + e^- + \nu$ may have different probabilities for the long-lived component.

We consider first the following decay channel of the $K$-particle:

$$K^0 \to e^- + \pi^+ + \bar{\nu}.$$  \hspace{1cm} (5.76)

The final product may be in states with either parity $= +1$ or parity $= -1$. Let us denote the matrix elements for the decay process into these two types of states by $f_1$ and $f_2$. Similarly, we denote the matrix elements for

$$K^0 \to e^- + \pi^- + \nu$$  \hspace{1cm} (5.77)

with the final state having parity $= +1$ and parity $= -1$ by $g_1$ and $g_2$.

By using the CPT theorem and Equation (4.14), the corresponding matrix elements for the decay of $\bar{K}$,

$$\bar{K}^0 \to e^- + \pi^- + \nu,$$  \hspace{1cm} (5.78)

are related to that of (5.76). These elements are $f_1^*$ and $-f_2^*$. Similarly the matrix elements for

$$\bar{K}^0 \to e^- + \pi^- + \bar{\nu}$$  \hspace{1cm} (5.79)

are $g_1^*$ and $-g_2^*$. Let $\psi_+$ represent the long-lived component $K$, of the $K$-particle. The matrix elements for the decay of $K$,

$$K_+ \to e^- + \pi^+ + \bar{\nu},$$  \hspace{1cm} (5.80)

into the two different final parity states are proportional to $pf_1 + qg_1^*$ and $pf_2 - qg_2^*$, respectively, while the corresponding elements for

$$K_+ \to e^- + \pi^- + \nu$$  \hspace{1cm} (5.81)

are proportional to $pg_1 + qf_1^*$ and $pg_2 - qf_2^*$. The branching ratio $r$ for the decay of $K_+$ into $e^- + \pi^+ + \bar{\nu}$ and $e^- + \pi^- + \nu$ is therefore,

$$r = \frac{|pf_1 + qg_1^*|^2 + |pf_2 - qg_2^*|^2}{|pg_1 + qf_1^*|^2 + |pg_2 - qf_2^*|^2}. \hspace{1cm} (5.82)$$

A detection of $\gamma \neq 0$ would establish the non-invariance of $K^0$ decay under time reversal. However, from (5.82), we see that if $|p| = |q|$ then $r = 0$. Also, if $a = 0$, the two eigenstates $\psi_+$, $\psi_-$ are orthogonal and $|p| = |q|$. (This is the case if the mass operator $M$ is negligible.) In this case the decays of long-lived components into charge conjugate channels such as $\pi^+ + e^- + \nu$ and $\pi^- + e^- + \bar{\nu}$ are equally probable. Furthermore we recall that because the strong interaction conserves $I_z$, the behavior of $|K^0\rangle$ under a charge conjugation operator cannot be determined within a phase factor $e^{i\alpha \theta}$. If $\psi_+$ is orthogonal to $\psi_-$ by choosing this phase factor to be that of $(p/q)$ these two eigenstates $\psi_+$ and $\psi_-$ can be made to be identical with $|K_+\rangle$ and $|K_\pm\rangle$, Equation (5.55). From experimental results on the two lifetimes of $\psi_+$ and $\psi_-$ we know that $\alpha^2 < 0.04$. Thus the branching ratio $r$ may be quite small even though time reversal may not be conserved.
As a final remark we note that because of the largeness of phase space volume for $2\pi$ decay it might be expected that in the $\Gamma$ matrix [Equation (5.63)] only the matrix elements due to the decay into a certain $2\pi$ mode are of dominant importance. If in the calculation of the mass operator $M$ the virtual processes via the same $2\pi$ mode give also the dominant contributions, then we should expect without further assumptions that the lifetime ratio $\gamma_+ / \gamma_-$ should be large; that the short-lived one $\psi_-$ and the long-lived one $\psi_+$ should be almost identical with $K_1$ and $K_2$ respectively; and that $\psi_-$ should decay mostly into that $2\pi$ mode while $\psi_+$ should decay dominantly to other modes such as $\pi^+ + e^+ + \bar{\nu}$, etc.
VI. A TWO-COMPONENT THEORY OF THE NEUTRINO\textsuperscript{37}

The various experimental results on nonconservation of parity and charge conjugation in $\beta$ decay, $\pi$ decay, and $\mu$ decay can be expressed in a particularly simple and appealing way by using a two-component theory of the neutrino.

1. The Neutrino Field

Consider first the Dirac equation for a free spin $\frac{1}{2}$ particle with zero mass. Because of the absence of the mass term one needs only three anticommuting Hermitian matrices. Thus the neutrino can be represented by a spinor function $\phi_r$ which has only two components. The Dirac equation for $\phi_r$ can be written as ($\hbar=c=1$)

$$ (\sigma \cdot p) \phi_r = i\dot{\phi}_r, $$

(6.1)

where $\sigma_1, \sigma_2, \sigma_3$ are the usual $2\times2$ Pauli matrices. The relativistic invariance of this equation for proper Lorentz transformations (i.e., Lorentz transformations without space inversion and time inversion) is well known. In particular for the space rotations through an angle $\theta$ around, say, the $z$ axis, the wave function transforms in the following way:

$$ \phi_r \rightarrow \exp(-i\sigma_3\theta/2) \phi_r. $$

The $\sigma$ matrices are therefore the spin matrices for the neutrino. For a state with a definite momentum $p$, the energy and the spin along $p$ (defined to be the helicity $\mathcal{H}$) are given respectively by

$$ H = (\sigma \cdot p) \quad \text{and} \quad \mathcal{H} = (\sigma \cdot p) / |p|. $$

(6.2)

They are therefore related by

$$ H = |p| \sigma_p. $$

(6.3)

In the $\epsilon$ number theory, for a given momentum the particle therefore has two states: a state with positive energy, and with $\frac{1}{2}$ as the spin component along $p$, and a state with negative energy and with $-\frac{1}{2}$ as the spin component along $p$.

It is easy to see that in a hole theory of such particles, the spin of a neutrino (defined to be a particle in the positive energy state) is always parallel to its momentum.

\textsuperscript{37} The possibility of a two-component relativistic theory of a spin $\frac{1}{2}$ particle was first discussed by H. Weyl [Z. Physik 56, 330 (1929)]. However, in such a theory parity is not manifestly conserved; therefore, in the past it was always rejected. (Cf. W. Pauli, Handbuch der Physik, Verlag Julius Springer, Berlin, 1933, Vol. 24, pp. 226-7.) The possible use of this two-component theory for expressing the nonconservation property of parity in neutrino processes was independently proposed and considered by T.D. Lee and C.N. Yang, Phys. Rev. 105, 1671 (1957); A. Salam, Nuovo cimento 5, 299 (1957); and L. Landau, Nuclear Phys. 3, 127 (1957).
while the spin of an antineutrino (defined to be a hole in the negative energy state) is 
always antiparallel to its momentum (i.e., the momentum of the antineutrino). Many 
of the experimental implications on nonconservation of parity and charge conjugation 
can be directly related to this correlation between the spin and the momentum of a neutrino. With the usual (right-handed) conventions which we adopt throughout this paper, the spin and the velocity of the neutrino represent the spiral motion of a right-handed screw ($3\mathcal{C}_r = +1$), while the spin and the velocity of the antineutrino represent the spiral motion of a left-handed screw ($3\mathcal{C}_r = -1$).

We shall now discuss some general properties of this neutrino field under the further assumption that the law of conservation of leptons is valid.\footnote{If the law of conservation of leptons is not valid then the mass of a two-component neutrino is, in general, not zero. In a special case even parity may still be conserved. See footnote 39 and K.M. Case, Phys. Rev. 107, 307 (1957). Cf. also A. Salam, Nuovo cimento 5, 299 (1957).}

A. The mass of the neutrino and the antineutrino in this theory is necessarily zero. This is true for the physical mass even with the inclusion of all interactions. To see this, one need only observe that all the one-particle physical states consisting of one neutrino (or one antineutrino) must belong to a representation of the inhomogeneous proper Lorentz group identical with the representation to which the free neutrino states discussed above belong. For such a representation to exist at all the mass must be zero.

B. That the use of such a theory fits naturally into the nonconservation property of parity is well known. We see it also in the following way: Under a space inversion $P$, one inverts the momentum of a neutrino but not its spin direction. Since in this theory the two are always parallel, the operator $P$ applied to a neutrino state leads to a nonexisting state. Consequently the theory is not invariant under space inversion.

C. By the same reasoning one concludes that the theory is also not invariant under charge conjugation $C$ which changes a particle into its antiparticle but does not change its spin direction or momentum.

D. It is possible, however, for the theory to be invariant under the operation $CP$, as this operation changes a neutrino into an antineutrino and simultaneously reverses its momentum while keeping the spin direction fixed. By the $CPT$ theorem it follows that the theory can be invariant under time reversal $T$.

For the free neutrino field, as described by (6.1), one can prove that the theory is indeed invariant under time reversal and under $CP$.

2. $\beta$ Decay

To discuss the $\beta$ decay phenomena with the two-component neutrino theory and to compare the present results with those calculated previously, it is convenient to indicate how one can use the conventional four-component formalism of the neutrino (with nonconservation of parity) and obtain the same results as the present theory.

We start from Equation (6.1) and enlarge the matrices by the following definitions: $(1\equiv 2 \times 2$ unit matrix)
\[ \alpha = \begin{pmatrix} \sigma & 0 \\ 0 & -\sigma \end{pmatrix}, \quad \beta = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad (6.4) \]

\[ \psi_\nu = \begin{pmatrix} \phi_\nu \\ \phi^\nu \end{pmatrix}, \quad (6.5a) \]

\[ \gamma = -i\beta\alpha, \quad \gamma_4 \equiv \beta, \quad \gamma_5 \equiv \gamma_1\gamma_2\gamma_3\gamma_4 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}. \quad (6.6) \]

An immediate consequence of these definitions is
\[ \gamma_5 \psi_\nu = -\psi_\nu. \quad (6.7a) \]

The free neutrino part of the Lagrangian is, as usual,
\[ L_\nu = \psi_\nu^\dagger \gamma_\mu \left( \gamma_5 \frac{\partial}{\partial x_\mu} \right) \psi_\nu, \quad (6.8) \]

where \( \psi_\nu^\dagger = \text{Hermitian conjugate of } \psi_\nu \). The most general interaction Lagrangian with no derivative couplings for the process
\[ n \rightarrow p + e^- + \bar{\nu} \quad (6.9a) \]

is exactly as usual; namely, it is the sum of the usual \( S, V, T, A, \) and \( P \) couplings:
\[ + L_{\text{int}} = -H_{\text{int}} = \sum_i -\mathcal{G}_i (\psi_\nu^\dagger O_i \psi_\nu) (\psi_\nu^\dagger O_i \psi_\nu) \quad (6.10a) \]

where \( i \) runs over \( S, V, T, A, \) and \( P \); and \( O_S, O_V, \) etc. are given in Equation (5.2).

It is not difficult to prove that Equations (6.5a) and (6.7a) are consistent with a relativistical theory even in the presence of the interaction (6.10a). Another way of proving this is to start from the conventional theory of the neutrino with the interaction Hamiltonian given in Equation (5.1) and observe that when
\[ C_S = -C_S' = (\frac{1}{2}) \mathcal{G}_S \quad \text{and} \quad C_V = -C_V' = (\frac{1}{2}) \mathcal{G}_V, \quad \text{etc.}, \quad (6.11a) \]

the neutrino field \( \psi_\nu \) always appears in interactions in the combinations \( (1 - \gamma_5) \psi_\nu \). In the explicit representation we have adopted above this means that only the first two components of \( \psi_\nu \) contribute to the interaction. All calculations using the conventional theory of the neutrino with the Hamiltonian Equation (5.1) concerning \( \beta \) decay therefore give the same result as the present theory if we take the choice of constants (6.11a). There exists, however, the possibility that in the decay of the neutron a neutrino (defined to be a right-handed screw) is emitted,
\[ n \rightarrow p + e^- + \bar{\nu}. \quad (6.9b) \]

The corresponding general form (not including derivatives of the fields) of the Hamiltonian is
\[ H_{\text{int}} = \sum \mathcal{G}_i' (\psi_\nu^\dagger O_i \psi_\nu) (\psi_\nu^\dagger O_i \psi_\nu') \quad (6.10b) \]

where \( O_i \) has been defined in Equation (5.2). The field \( \psi_\nu' \) is a four-component spinor defined in terms of the two-component neutrino field \( \phi_\nu \) by
\[ \psi_\nu' = \begin{pmatrix} 0 \\ \sigma_2 \phi_\nu^\dagger \end{pmatrix}. \quad (6.5b) \]
From Equation (6.6) we see that
\[ \gamma_s \psi_\nu = 0 \psi_\nu' . \] (6.7b)

It can be shown that (6.5b) and (6.7b) are consistent with a relativistic theory even in the presence of interaction (6.10b). It can also be proved that one can use again the Hamiltonian Equation (5.1) for the conventional theory of the neutrino with the choice of the coupling constants
\[ \frac{1}{2} \Theta_\nu' = C_\nu = C_\nu' , \quad \frac{1}{2} \Theta_\bar{\nu} = C_{\bar{\nu}} = C_{\bar{\nu}}' , \quad \text{etc.} \] (6.11b)

and obtain the same result as the present theory.

The two possible choices (6.11a) and (6.11b) depend on whether, in the $\beta$ decay of the neutron process, (6.9a) or (6.9b) prevails, i.e., whether an antineutrino ($\mathcal{C}_s = -1$, a left-handed screw) or a neutrino ($\mathcal{C}_s = +1$, a right-handed screw) is emitted. We shall see that experimentally it will be easy to decide which of the two choices is appropriate. [In this section we do not consider the possibility of the simultaneous presence of both (6.9a) and (6.9b).]39

A. ANGULAR DISTRIBUTION OF $\beta$-RAYS FROM POLARIZED NUCLEI

The experimental results of Wu et al. show that in the decay of
\[ ^{60}\text{Co} \rightarrow ^{60}\text{Ni} + \varepsilon + \bar{\nu} , \quad J = 5 \rightarrow J = 4 , \] (6.12)
the angular distribution of $\varepsilon$ is
\[ 1 + \alpha \cos \theta \] (6.13)
with
\[ \alpha \equiv -0.7 \frac{(J_z)}{J} \] (6.14)
and
\[ \theta = \angle (J, p_\varepsilon) . \]
This result is consistent with the coupling constants assignment
\[ C_{\bar{\nu}}' = -C_\nu , \quad C_{\nu}' = -C_{\bar{\nu}} \equiv 0 . \] (6.15)

By comparing (6.15) with (6.11a) and (6.11b) we see that in the $\beta^-$ decay a $\bar{\nu}$ (with a left-hand helicity, $\mathcal{C}_s = -1$) is emitted, while in the $\beta^+$ decay a $\nu$ (with a right-hand helicity, $\mathcal{C}_s = +1$) is emitted.

39If the conservation law of leptons is valid then (6.9a) and (6.9b) cannot be simultaneously present. It is of interest to note that if reactions (6.9a) and (6.9b) were simultaneously present with exactly equal amplitude, the angular distribution of $\beta$-rays from polarized nuclei, for example, would appear to be symmetrical. In such a case the theory is identical with the conventional Majorana theory of the neutrino with a parity conserving Hamiltonian. This, of course, is not the case that occurs in nature. The general relationship between the present two-component theory and the Majorana theory with a parity nonconserving Hamiltonian and, possibly, nonzero mass has recently been investigated by K. Case, Phys. Rev. 107, 307 (1957).
B. LONGITUDINAL POLARIZATION OF $\beta$-RAYS

As remarked before, if parity is not conserved the electrons can be longitudinally polarized. We shall see that with the two-component theory of the neutrino all $\beta$ emitters can be used as almost perfect polarizers for the $\beta$-rays. The degree of polarization depends on $v/c$ of the $\beta$-rays. Let us consider a coupling term between $e^-$ and $\nu$,

$$\psi_{e^+}^* O_i \psi_{\nu},$$

with $O_i$ given in Equation (5.2). By using Equation (6.7a), together with the commutation relations

$$O_i \gamma_5 = \pm \gamma_5 O_i,$$

with the plus sign for $i = V, A$, and the minus sign for $i = S, T, P$, we obtain

$$\psi_{e^+}^* O_i \psi_{\nu} = \psi_{e^+}^* O_i \psi_{\nu} \quad \text{if} \quad i = V, A;$$

$$= \psi_{e^+}^* O_i \psi_{\nu} \quad \text{if} \quad i = S, T, P; \quad (6.16)$$

where

$$\psi_{e^+} = (\frac{1}{2}) (1 - \gamma_5) \psi_{e^+} \quad \text{and} \quad \psi_{\nu} = (\frac{1}{2}) (1 + \gamma_5) \psi_{\nu}. \quad (6.17)$$

If the electron is extremely relativistic, $v/c = 1$, then $\psi_{e^+}$ and $\psi_{\nu}$ are both eigenstates of the free Hamiltonian of the electron, provided the mass term is neglected. In this case if the coupling (6.16) is $S$ and/or $T$ and/or $P$ the electron will be $100\%$ polarized with its spin antiparallel to its momentum (i.e., with a left-handed helicity), while if the coupling is $V$ and/or $A$ the electron is also $100\%$ polarized but with its spin parallel to its momentum (i.e., with a right-handed helicity).

In general for relativistic or nonrelativistic electrons the term $(1+\gamma_5)/2$ acts as a projection operator for the longitudinal polarization. The helicity $\mathcal{H}$ can be calculated as

$$\mathcal{H}_{e^+} = (\sigma \cdot \hat{p}_{e^+})_{AV} = \pm v/c \quad (6.18)$$

where $\hat{p}_{e^+}$ is a unit vector along the momentum of $e^-$. In Equation (6.18) the plus sign is for $V, A$ couplings and the minus sign for $S, T, P$ couplings. Thus we see that in

$$n \rightarrow p + e^- + \bar{\nu}$$

if vector and axial vector couplings are absent the electrons will be longitudinally polarized with

$$\mathcal{H}_{e^+} = -v/c. \quad (6.19)$$

Equation (6.19) is true for any $\beta$ emitter independent of whether the nuclei are polarized or not, and is independent of whether the $\beta$ decay is allowed or forbidden. The possible deviation from (6.19) can then be used as a measure for the strengths of vector and axial vector coupling constants. In deriving Equations (6.18) and (6.19) we have not included possible depolarization effects due to a Coulomb field. However,
if \( v/c = 1 \) then it is easy to see that these results of longitudinal polarization cannot be effected by any Coulomb field.\(^{49}\)

Similarly for any \( \beta^- \) emission,

\[
p \rightarrow n + e^- + \nu ,
\]

the positions are longitudinally polarized with

\[
3C_{\epsilon_z} = v/c
\]

(6.20)

provided \( V \) and \( A \) couplings are zero.

Because of these properties it is possible to understand in a simple way that in the decay of \( \text{Co}^{60} \), Equation (6.12), if \( \bar{\nu} \) is emitted the electrons are emitted predominantly antiparallel to the spin direction of \( \text{Co}^{60} \) [i.e., \( \alpha < 0 \) in Equation (6.14)]. To understand the sign of \( \alpha \), let us neglect the \( A \) coupling, and consider the special case with

\[
(J_z)_{\text{Co}^{60}} = 5 \rightarrow (J_z)_{\text{Ni}^{60}} = 4
\]

in the \( \text{Co}^{60} \) decay. The \( e^- \) and \( \bar{\nu} \) emitted are both left-handed particles (i.e., spin antiparallel to momentum). Take the particular case that \( p_e, p_{\bar{\nu}} \) are all parallel to the \( \pm z \) axes. In this case the orbital angular momentum along the \( z \) axis is zero. In order to conserve \( J_z \), both \( e^- \) and \( \bar{\nu} \) must be emitted predominantly parallel to each other with both of their momenta \( p_e \) and \( p_{\bar{\nu}} \) along the \( -z \) axis (cf. Figure 2). By using (6.19), it is easy to see that in this case with \( C_{\alpha} = 0 \) the relative probabilities are

\[
1 - (v/c) \quad \text{for} \quad \cos \theta = +1
\]

and

\[
1 + (v/c) \quad \text{for} \quad \cos \theta = -1 .
\]

According to Equations (5.11) and (6.11a) the complete formula of \( \alpha \) for

\(^{49}\)This may be proved in the following way. Consider the motion of a charged particle in a Coulomb field \( U \),

\[
(\alpha \cdot p + \beta m + U)\psi = E\psi .
\]

Except for the \( \beta m \) term the Hamiltonian commutes with \( (1 \pm \gamma_z) \). Thus if the final expression for \( (\sigma \cdot \hat{p}) \) is expanded in a power series of \( (m/E) \), the term with zeroth power in \( (m/E) \) is independent of the presence of \( U \); i.e.,

\[
(\sigma \cdot \hat{p})_{\epsilon_z} = +1 + o(m/E)
\]

for \( V, A \) couplings and

\[
(\sigma \cdot \hat{p})_{\epsilon_z} = -1 + o(m/E)
\]

for \( S, A, P \) couplings.
\[ J \to J(\text{no}) \] (6.21)
is
\[ \alpha = \frac{J}{J} \beta, \]
\[ \beta = -\frac{\nu}{c} \frac{|C_p|^2 - |C_a|^2 + (2Ze^2/\hbar c)p\text{Im}(C_aC^*_p)}{|C_p|^2 + |C_a|^2}. \] (6.22)

In (6.22) the Fierz term is put to be zero. The minus sign in the coefficient of \(|C_a|^2\) is of course due to the fact that \(\nu\) is emitted with a right-hand helicity via the axial vector coupling.

By a similar argument, for the corresponding simple case of \(\beta\) decay in
\[ J \to J+1(\text{no}) \] (6.23)
the \((\nu, \bar{\nu})\) system should carry a \((\Delta J_z) = -1\). Thus the electrons are expected to be emitted predominantly parallel to \(p_\nu\) and both of these momenta are parallel to the spin direction of the nuclei. The general formula for (6.23) is
\[ \alpha = -\frac{J_z}{J+1} \beta \] (6.24)
with \(\beta\) given by (6.22). The difference of sign in (6.24) and (6.21) is completely expected.

In the decay
\[ J \to J(\text{no}) \] (6.25)
let us first consider the contribution due to the tensor coupling in the Gamow-Teller matrix element alone. Even if one takes the special case of the completely polarized nuclei \(J_z = J\) the change of \(z\) component nuclear angular momentum can be 0 or 1.

In the first case of \(\Delta J_z = 0\) no asymmetry is present. In the second case, the \(\nu\) and \(\bar{\nu}\) should carry a \(\Delta J_z = +1\) which causes the asymmetry parameter \(\alpha\) to be negative, \(\alpha < 0\). Thus we expect in the transition (6.25) if \(M_\nu = 0\) and \(C_a = 0\) the sign of \(\alpha\) should be negative but with its magnitude greatly reduced. The general formula for \(J \to J(\text{no})\) is
\[ \alpha = \frac{J_z}{J(J+1)} \beta + \frac{J_z}{\sqrt{J(J+1)}} \beta', \]
\[ \beta' = -Re \left[ C_{gs}^* C_{gs} - C_{gs}^* C_a + i \frac{Ze^2}{\hbar c} p (C_{gs}^* C_a - C_{gs}^* C_{gs}) \right] \frac{\nu}{c} \frac{4M_p \cdot M_{G_T}}{\xi + (\xi b/W)} \] (6.26)
with \(\xi\) and \(b\) given in Equations (5.4) and (5.8).

Similarly it is easy to see that for \(\beta\) decay the asymmetry parameter due to tensor coupling (or due to the axial vector coupling term alone) must change its sign. The complete formulas can be easily obtained from Equations (5.11) to (5.13).
3. Capture Cross Sections for the Neutrino

An experiment such as the one being carried out by Cowan et al.\(^4\) measures the cross section for neutrino absorption, which can be calculated in both the present theory and the usual theory. Now one determines the magnitude of the $\beta$ coupling constants to give the observed lifetimes of nuclei against $\beta$ decay. The calculated value of the cross section turns out then to be \textit{twice as great} in the present theory as in the usual theory using the four-component theory and conservation of parity. This follows from the following simple reasoning: The neutrino flux is an experimental quantity independent of the theory. If the neutrinos in a given direction have only one spin state instead of the usual two, by a detailed balancing argument they must have a cross section for absorption twice as great as the usual ones. Actually from the experiments of Wu et al.\(^1\) one expects the neutrino emitted to be longitudinally polarized. Thus an increment of cross section is expected if we use the hermiticity property of the $H_{\text{weak}}$. This effect should be present even if the neutrino is described by a four-component theory with parity nonconservation.

4. $\pi$ Decay

In the decay of $\pi$-mesons at rest let us consider the component of angular momentum along the direction $p_\pi$, the momentum of the $\mu$-meson. The orbital angular momentum contributes nothing to this component. The $\mu$ spin component is therefore completely determined (irrespective of its total spin) by the spin component of the $\nu$ or $\bar{\nu}$. There are then two possibilities: (i)

\[ \pi^+ \rightarrow \mu^+ + \nu, \text{ } \mu^+ \text{ spin along } p_\mu = +\frac{1}{2}, \]

\[ \pi^- \rightarrow \mu^- + \bar{\nu}, \text{ } \mu^- \text{ spin along } p_\mu = -\frac{1}{2}; \]

or (ii)

\[ \pi^+ \rightarrow \mu^- + \bar{\nu}, \text{ } \mu^- \text{ spin along } p_\mu = -\frac{1}{2}, \]

\[ \pi^- \rightarrow \mu^+ + \nu, \text{ } \mu^+ \text{ spin along } p_\mu = +\frac{1}{2}. \]

In each case the $\mu$-mesons with fixed $p_\mu$ form a polarized beam. Furthermore, the polarization is complete (i.e., in a pure state). In this theory of the neutrino the $\pi$-$\mu$ decay is then a perfect polarizer of the $\mu$-meson, and offers a natural way to measure the spin and the magnetic moment of the $\mu$-meson. (It turns out that the $\mu$-$\epsilon$ decay can serve as a good analyser, as we shall discuss in the next section.)

The choice of the two possibilities (6.27) and (6.28) will be further discussed in Chapter VII.

5. $\mu$ Decay

For the $\mu^-\epsilon^-$ decay the process can be

$$\mu^-\rightarrow\epsilon^-+\nu+\bar{\nu}, \quad (6.29)$$

or

$$\mu^-\rightarrow\epsilon^-+2\nu, \quad (6.30)$$

or

$$\mu^-\rightarrow\epsilon^-+2\bar{\nu}. \quad (6.31)$$

Consider process (6.29) first. The decay interaction (without derivative coupling) can be written with the notations defined in Equation (5.2):*2

$$H_{\text{int}} = \sum_i G_i (\psi^* \gamma_i \psi_\nu) (\psi^* \gamma_i \psi_\mu). \quad (6.32)$$

We have assumed in writing (6.32) that the spin of the $\mu$-meson is $\frac{1}{2}$.

Because of the subsidiary condition (6.7a) satisfied by the neutrino field $\psi_\nu$, the $S$-coupling term in the Hamiltonian (6.32) gives a result identical to that of the $P$-coupling term; the $V$-coupling term is the same as the $A$-coupling term; and the $T$-coupling term is identically zero. Thus, in (6.32) there are only two independent constants,

$$g_1 \equiv G_N - G_\rho \quad \text{and} \quad g_2 \equiv G_V + G_A. \quad (6.33)$$

The electron (or positron) emitted from a $\mu$-meson decay at rest will be longitudinally polarized with a helicity $\mathcal{C}$ given by

$$\mathcal{C}_\epsilon = -\xi \quad \text{and} \quad \mathcal{C}_\nu = +\xi \quad (6.34)$$

where

$$\xi = (|g_1|^2 - 4|g_2|^2)(|g_1|^2 + 4|g_2|^2)^{-1} \quad (6.35)$$

and the helicity $\mathcal{C}$ is defined by

$$\mathcal{C} = (\sigma \cdot \mathbf{p})/|\mathbf{p}| \quad (6.2)$$

with $\sigma$ and $\mathbf{p}$ the spin and momentum vectors of the electron (or positron). Equation (6.34) is independent of the polarization state of the $\mu$-meson.

For a $\mu^-$ at rest with spin completely polarized, the normalized electron distribution is given by

---

*In reference 23 the Hamiltonian for $\mu$ decay is written as

$$H = \sum_{i=A} G_i (\psi^*_i \gamma_i \psi_\mu).$$

The $f_\Delta$ and $f_\nu$ are related to $G_i$ of (6.32) by

$$(\frac{1}{2})(G_N - G_\rho) = f_\Delta + f_\nu \quad \text{and} \quad G_V + G_A = f_\Delta - f_\nu.$$ 

It is also possible to write the Hamiltonian for $\mu$ decay in the form

$$H = \sum_i G'_i (\psi^*_i \gamma_i \psi'_\nu) (\psi^*_i \gamma_i \psi'_\mu)$$

where $\psi'_\nu$ is given by (6.56). The $G'_i$ are related to $G_i$ by

$$(\frac{1}{2})(G_N - G_\rho) = G'_V + G'_A \quad \text{and} \quad G_V + G_A = - (\frac{1}{2})(G_N - G_\rho).$$
\[(dN)_e = 2x^2[(3-2x) + \xi \cos \theta(1-2x)] dx \, d\Omega(4\pi)^{-1}\]  \hspace{1cm} (6.36)

where

\[x = \rho/(\text{maximum electron momentum}),\]

\[\theta = \text{angle between the electron momentum and the spin direction},\]

\[d\Omega = \text{solid angle of electron momentum},\]

and \(\xi\) is given by (6.35). The corresponding distribution of \(e^+\) from a completely polarized \(\mu^+\) at rest is

\[(dN)_e = 2x^2[(3-2x) - \xi \cos \theta(1-2x)] dx \, d\Omega(4\pi)^{-1}.\]  \hspace{1cm} (6.37)

The decay probability per unit time is \((\hbar = \epsilon = 1)\)

\[\lambda = m_\mu^2(|g_1|^2 + 4|g_2|^2)(3 \times 2^{-1} \pi^2).\]  \hspace{1cm} (6.38)

The energy spectrum of the electron (or positron) is

\[dN = 2x^2(3-2x) dx.\]  \hspace{1cm} (6.39)

The spectrum (6.39) is characterized by a Michel parameter\(^{41}\) \(\rho = \frac{1}{4}\) which is not inconsistent with but seems to be slightly higher than the present experimental value\(^{44}\) \(\rho = 0.68\).

Integration of (6.36) and (6.37) over the energy of \(e^\pm\) gives for the over-all angular distribution

\[(dN)_e = [1 \pm (\frac{1}{2})\xi] d\Omega(4\pi)^{-1}.\]  \hspace{1cm} (6.40)

The mass of the electron (or positron) is neglected in all the above formulas (6.34) to (6.40).

That in (6.37) the angular distribution depends sensitively on the energy of the electron can be understood in a simple way. By using (6.16) to (6.18), we expect for the helicity of \(e^-\) [cf. (6.35)]

\[\mathcal{K}_{e^-} = -1 \quad \text{if only } g_1 \neq 0\]

and

\[\mathcal{K}_{e^-} = +1 \quad \text{if only } g_2 \neq 0.\]  \hspace{1cm} (6.41)

Consequently, there is no interference term between \(g_1\) and \(g_2\).

Let us first discuss the case that only \(g_1 \neq 0\) (\(g_2 = 0\)). The \(e^+\) emitted is of left-hand helicity. Consider the special case that \(p_e, p_\pi, p_\mu\), and \(\sigma\) are all along the \(+z\) or \(-z\) axis. From conservation of angular momentum along the \(z\) axis it is easy to show that if \(\sigma_\mu// +z\) axis then for \(x = 1\),

\[p_e// -z\text{ axis}\]  \hspace{1cm} (6.42)

and for \(x < \frac{1}{2}\),

\[p_e// +z\text{ axis} \quad (g_1 \neq 0, g_2 = 0).\]  \hspace{1cm} (6.43)


\(^{42}\)C.P. Sargent, M. Rinchart, L.M. Lederman, and K.C. Rogers, *Phys. Rev.* 99, 885 (1955). These authors give \(\rho = 0.68 \pm 0.10\). More recent measurements by L. Rosenson (*Phys. Rev.*, in press) and by K. Crowe, *Bull. Am. Phys. Soc.* 2, 206 (1957), give the same value for \(\rho\) but with a smaller error. A slight deviation of \(\rho\) value from 0.75 may indicate that there is a possible "non-local" effect in the \(\mu\) decay Lagrangian.
This can be seen directly by inspection of Figure 3 (case A₁ and case A₂). Furthermore for \( x = 1 \) all three momenta \( \mathbf{p}_e, \mathbf{p}_\mu, \) and \( \mathbf{p}_\nu \) must be collinear. Thus, from (6.42), the corresponding angular distribution must be of the form \( 1 - \cos \theta \) (with \( x = 1 \) and \( g_z = 0 \)), which means that the asymmetry is maximum. For other values of \( x(<1) \) these three momenta may not be collinear and the asymmetry does not attain its maximum value. A comparison between (6.42) and (6.43) explains the energy dependence \( (1 - 2x) \) in the \( \cos \theta \) term in (6.37). In an entirely similar way one can apply the above considerations to the case that \( g_1 = 0 \) and \( g_2 \neq 0 \) (cases B₁ and B₂ of Figure 3).

Experimentally, the angular distribution of \( \epsilon^\mu \) has been measured\(^{12,25}\) with respect to the momentum of the \( \mu^- \) meson from \( \pi^- \) decay. The experimental results seem to agree quite well with the distribution function (6.37). The results for \( \mu^- \) stopped in carbon were discussed in Chapter V, section 5. From Equation (5.33), it can be concluded that the parameter \( \xi \), (6.35), must lie within the limits

\[
1 > |\xi| > 0.78,
\]

(6.44)

The algebraic sign of \( \xi \) depends on whether in \( \pi^- \) decay the helicity of \( \mu^- \) is \( +1 \) or \( -1 \). Equation (6.44) gives only a lower limit of \( \xi \). The actual value depends on the degree of depolarization of \( \mu^- \). The precise value and sign of \( \xi \) can be obtained more directly by a measurement of the helicity for \( \epsilon^\mu \) from \( \mu^- \) decay, Equation (6.34). This point will be further discussed in connection with the law of conservation of leptons in Chapter VII.

If in the \( \mu^- \) decay process

\[
\mu \rightarrow e^- + 2\nu
\]

(6.30)
or

\[
\mu \rightarrow e^- + 2\overline{\nu}
\]

(6.31)
prevalls, the corresponding energy spectrum is characterized by a Michel parameter $\rho = 0$ which is not consistent with experiments.$^{44}$

6. Remarks

In the above sections we have seen that the various effects due to nonconservation of parity and charge conjugation in $\beta$ decay, $\pi$ decay, and $\mu$ decay can be conveniently described by the use of a two-component theory of the neutrino. Nevertheless, as we have noticed before (cf. footnote 39) the mere use of a two-component theory does not preclude, for example, the possibility that in addition to

$$n \rightarrow p + e^- + \bar{\nu}$$  \hspace{1cm} (6.9a)

we may also have

$$n \rightarrow p + e^- + \nu.$$  \hspace{1cm} (6.9b)

From the experimental results on the slowness of the rate for double $\beta$ decay processes and the largeness of asymmetry in the $\beta$ angular distribution from polarized nuclei, we know that reaction (6.9b), if it exists at all, must be described by a much smaller coupling constant than that of reaction (6.9a). Recently K. Case$^{45}$ was able to show that by using the Majorana theory with a Hamiltonian which does not conserve parity it is possible to generalize the equation of motion [Equation (6.1)] for the neutrino field and to construct a two-component theory with, possibly,

$$m_\nu \neq 0.$$  

In the special case that $m_\nu = 0$, his generalization reduces to the present two-component theory discussed in this chapter. However, in the general case, if the mass of the neutrino $m_\nu \neq 0$ then the rate of double $\beta$ decay process cannot attain its minimum value and the asymmetry of the $\beta$ angular distribution from polarized nuclei cannot reach its maximum value. Yet experimentally, the mass of the neutrino $m_\nu \geq 0$, the rate of double $\beta$ decay process $\geq 0$, and the asymmetry of the $\beta$ angular distribution $\geq$ its maximum value. These three facts seem to be strongly suggestive of the possible existence of a law of conservation of leptons.$^{46}$ It can be shown easily that the existence of a conservation law of leptons together with the use of a two-component theory of the neutrino necessitates (i) $m_\nu = 0$, (ii) rate of double $\beta$ decay $=$ its minimum value, and (iii) parity must be nonconserved and the observed asymmetry due to such nonconservation can attain its maximum value. In the next chapter we shall analyze in some detail the various consequences of such a conservation law of leptons.


$^{46}$The concept of a possible conservation law of leptons was first considered by E. Konopinski and H.M. Mahmoud, Phys. Rev. 92, 1045 (1953). Some discussions on analyzing the conservation law of leptons together with the use of a two-component theory of the neutrino are given in T.D. Lee and C.N. Yang, Phys. Rev. 105, 1671 (1957).
VII. POSSIBLE LAW OF CONSERVATION OF LEPTONS
AND THE UNIVERSAL FERMI INTERACTIONS

1. Law of Conservation of Leptons

The law of conservation of leptons states that to each particle it is possible to assign a leptonic number \( l \) (\( l \neq 0 \) for leptons) and the sum of leptonic numbers must be conserved in all reactions. From our previous discussions we know that \( \beta \) decay and \( \mu \) decay are represented by

\[
\begin{align*}
n & \rightarrow p + e^- + \bar{\nu} \\
\mu^- & \rightarrow e^- + \nu + \bar{\nu}.
\end{align*}
\]

and

Consequently, the assignments of \( l \) must be chosen as

\[
\begin{align*}
l = & \text{same (say, } l = -1) \text{ for } e^-, \nu, \text{ and } \mu^- , \\
l = & +1 \text{ for } e^+, \bar{\nu}, \text{ and } \mu^+ , \text{ and} \\
l = & 0 \text{ for } \pi, \gamma, K, \text{ and all heavy particles.}
\end{align*}
\]

The conservation law of leptons then necessitates for the \( \pi \) decay

\[
\pi^+ \rightarrow \mu^- + \bar{\nu} \text{ and } \pi^0 \rightarrow \mu^+ + \nu .
\]

Equation (7.4) implies that the helicities \( \mathcal{K} \) of \( \mu^- \) in the rest system of \( \pi^- \)-mesons are

\[
\mathcal{K}_{\mu^-} = -1 \text{ and } \mathcal{K}_{\mu^+} = +1
\]

where \( \mathcal{K} \) is defined to be

\[
\mathcal{K} = (\sigma \cdot p)/|p|
\]

with \( \sigma, p \) the spin and momentum vectors. The measurement of the helicities of \( \mu^- \) and \( \mu^+ \) from \( \pi^\pm \) decay can serve as a test for the validity of the conservation law of leptons.

Next we consider the helicities of \( e^+ \) in the \( \mu \) decay. As we have discussed before, the helicities of \( e^+ \) measured in the rest system of the \( \mu \)-meson are given by [see (6.34) and (6.35)]

\[
\mathcal{K}_{e^+} = -\xi \text{ and } \mathcal{K}_{e^-} = +\xi
\]

Throughout this chapter we use the two-component theory for the neutrino.

That the leptonic number \( l \) for all heavy particles can be chosen as zero is evident. Both pion and photon can be created singly, thus their leptonic numbers must both be zero. If the number of leptons is absolutely conserved, then, because of the existence of decay modes \( K_{\pi^2} \) and \( K_{\pi^0} \), the leptonic number \( l \) for \( K \)-mesons must also be zero.
where $\xi$ is related to the coupling constants for $\mu$ decay by (6.35) and (6.33). By comparing the theoretical angular distribution for $\epsilon$ with the observed value we have already found that

$$1 \geq |\xi| > 0.78.$$  

(6.44)

Now if the law of conservation of leptons is correct, by using (7.5) we conclude that

$$-1 \leq \xi < -0.78.$$  

(7.8)

The minus sign for $\xi$ is a consequence of the conservation law of leptons. Thus the measurement of the helicities of $\epsilon^-$ and $\epsilon^+$ from $\mu$ decay can also serve as a test of the validity of the conservation law of leptons.

The conservation law of leptons also has a direct effect on the helicities of $\mu^\pm$ from $K_{\mu \pm}^\pm$ decay. From the assignments of $l$ in (7.3), the $K_{\mu \pm}^\pm$ decay should be described by the reactions

$$K_{\mu \pm}^\pm \rightarrow \mu^\pm + \bar{\nu} \quad \text{and} \quad K_{\mu \pm}^\pm \rightarrow \mu^\mp + \nu.$$  

(7.9)

Thus if the spin of $K$ is zero the helicities of $\mu^\pm$ in the rest system of $K$ are respectively

$$\mathcal{H}_\mu = -1 \quad \text{and} \quad \mathcal{H}_{\mu^\pm} = +1.$$  

(7.10)

2. Universal Fermi Interactions

In this section we shall analyze the possibility of the so-called universal Fermi interactions by comparing the Hamiltonians for $\mu$ decay, $\beta$ decay, and $\mu$ capture processes.\(^{54}\) We represent these three processes by

$$H_1 = \sum G_i (\psi_e^\dagger O_1 \psi_\mu) (\psi_\mu^\dagger O_1 \psi_\mu) \quad \text{and} \quad \text{hermitian conjugate},$$

$$H_2 = \sum G_i (\psi_\mu^\dagger O_1 \psi_\mu) (\psi_\mu^\dagger O_1 \psi_\mu) \quad \text{and} \quad \text{hermitian conjugate},$$

$$H_3 = \sum G_i (\psi_e^\dagger O_1 \psi_\mu) (\psi_\mu^\dagger O_1 \psi_\mu) \quad \text{and} \quad \text{hermitian conjugate},$$

(7.11)

respectively.

From (7.8) and the lifetime of the $\mu$-meson we have for the coupling constants $G_i$ of $\mu$ decay (cf. Chapter VI, section 5)

\(^{54}\)The application of the conservation law of leptons to $K_{\mu \pm}^\pm$ decay is of interest because it gives a result opposite to that obtained by using the "attribute rule" proposed by R.G. Sachs [Phys. Rev. 99, 1573 (1955)]. Cf. also W.G. Holladay, Phys. Rev., in press.

\(^{55}\)For references to previous works on the various Fermi interactions see, e.g., E. Fermi, Elementary Particles, Yale University Press, 1951; L. Michel, Revs. Mod. Phys. 29, 159 (1957).

\(^{51}\)In the present discussion of universal Fermi interactions we group the spinor fields $\psi_e$, $\psi_\mu$, and $\psi_\nu$ in a particular form which is compatible with the idea of the conservation law of leptons. Had we grouped, for example, the $\mu$ decay interaction $H$ in a different way (cf. footnote 42), many of the following conclusions about the similarity between the $\beta$ decay coupling constants and the $\mu$ decay coupling constants would be changed. These changes can be easily obtained by using the relationship between $G_i$ and $G_i'$ given in footnote 42. (Compare especially $G_i'$ with $\delta_{ij}$.)

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\[ \eta = \frac{|G_v + G_S|}{|G_S - G_P|} > 2.3 \]  
(7.12)

and

\[ |G_S - G_P|^2 + 4|G_v + G_A|^2 = (7.5 \times 10^{-49} \text{ erg-cm}^3)^2. \]  
(7.13)

The right-hand side of (7.12) is only a lower limit. The actual value of \( \eta \) is probably much larger than this limit. For example, \( \eta \approx 5 \) if in the measurement of \( \pi^+ - \mu^- \epsilon \) decay 90% of the \( \mu \)-mesons stopped in carbon are polarized and \( \eta \rightarrow \infty \) if only 80% of the stopped \( \mu \)-mesons are polarized.

The corresponding values for the various coupling constants for \( \beta \) decay have been extensively studied.\(^{52}\) These results may be summarized\(^{53} \) as follows:\(^{52} \)

(i) From the absence of a Fierz term it can be concluded that

\[ \frac{|\Theta_S^* \Theta_V + \Theta_R^* \Theta_S|}{|\Theta_S^* \Theta_S + \Theta_R^* \Theta_R|} = 0.00 \pm 0.10 \]  
(7.14)

and

\[ \frac{|\Theta_A^* \Theta_T + \Theta_{\gamma}^* \Theta_A|}{|\Theta_A^* \Theta_A + \Theta_{\gamma}^* \Theta_{\gamma}|} = 0.00 \pm 0.04. \]  
(7.15)

(ii) From the \( \beta-\nu \) angular correlation experiment on \( \text{He}^8 \) it can be concluded that

\[ \left| \frac{\Theta_A}{\Theta_T} \right|^2 < \frac{1}{3}. \]  
(7.16)

(iii) From the \( \beta-\nu \) angular correlation experiment on \( \text{Ne}^{19} \) it can be concluded that

\[ \left| \frac{\Theta_V}{\Theta_S} \right|^2 < 1.4 \quad \text{if} \quad \Theta_A \approx 0 \]  
(7.17)

and

\[ \left| \frac{\Theta_V}{\Theta_S} \right|^2 < 3 \quad \text{if} \quad \left| \frac{\Theta_A}{\Theta_T} \right|^2 = \frac{1}{3}. \]  
(7.18)

(iv) From the \( \beta-\chi \) diagram it can be concluded that

\[ \frac{|\Theta_S|^2 + |\Theta_V|^2}{|\Theta_A|^2 + |\Theta_{\gamma}|^2} = 0.79. \]  
(7.19)

(v) From the \( \beta \) value of O\(^{14} \) and (7.19), the absolute magnitudes of \( |\Theta_S|^2 + |\Theta_V|^2 \) and \( |\Theta_A|^2 + |\Theta_{\gamma}|^2 \) are

\[ |\Theta_S|^2 + |\Theta_V|^2 = (2.0 \times 10^{-49} \text{ erg-cm}^3)^2 \]  
(7.20)

and

\[ |\Theta_A|^2 + |\Theta_{\gamma}|^2 = (2.5 \times 10^{-49} \text{ erg-cm}^3)^2. \]  
(7.21)

It should be noted that \( \Theta_i \) is related to \( C_i \) and \( C_i' \) in (5.1) by [cf. (6.11a)]

\[ \Theta_i = 2C_i = -2C_i'. \]  
(7.22)


\(^{53}\) These results, (7.14) to (7.21), on various \( \Theta_i \) for \( \beta \) decay were compiled by C.S. Wu. We are grateful to Professor Wu for her permission to quote these results here.
(vi) Thus, if the $\beta$ decay interaction is invariant under time reversal $T$, then

$$G_0 \equiv 0 \quad \text{and} \quad G_A \equiv 0.$$  \hspace{1cm} (7.23)

By comparing $G_i$ [(7.20), (7.21), and (7.23)] with $G_i$ [(7.12) and (7.13)] we see that the $\beta$ decay coupling constants $G_i$ are very different from the $\mu$ decay coupling constants $G_i$. In this case the idea of universal Fermi interactions seems to be difficult to maintain.\textsuperscript{51}

(vii) If $\beta$ decay interaction is not invariant under time reversal then the limits on $G_i$ are those given by (7.20), (7.21), and the inequalities (7.17) and (7.18). In this case the $\beta$ decay coupling constants may not be incompatible with the $\mu$ decay coupling constants.

In conclusion we wish to remark that to give a definitive statement on the universal Fermi interaction it is necessary to obtain a much sharper limit on $|G_v/G_0|^2$ than that in (7.17) and (7.18). Because of the nonconservation of parity this can now be obtained by a direct measurement on the helicities of $e^\pm$ from any $\beta$ transition that has a large Fermi matrix element $M_F$ [cf. (6.18)]. As we shall see in the next section, because of the nonconservation of parity it is also possible to measure the various $G_i$ for $\mu^-$ capture processes.

3. $\mu^-$ Capture Process\textsuperscript{54}

In this section we shall study in detail the capture of $\mu^-$ by nuclei,

$$\mu^- + p \rightarrow n + \nu .$$  \hspace{1cm} (7.24)

The Hamiltonian for this process is given by

$$H_3 = \sum_i G_i (\psi_\nu^* O_i \psi_\nu) (\psi_\mu^* O_i \psi_\mu)$$  \hspace{1cm} (7.25)

where $\psi_\nu$ is the two-component neutrino field. We list the following results for capture of $\mu^-$ in hydrogen.

(i) The rate for process (7.24) in hydrogen is

$$(1/\tau)_{\text{cap}} = \rho^2_{\nu} \xi / 2\pi a^3$$  \hspace{1cm} (7.26)

where

$$\xi = |G_\beta + G_\nu|^2 + 3 |G_A + G_\tau|^2.$$  \hspace{1cm} (7.27)

$\rho_{\nu}$ is the momentum of the neutrino and $a$ is the Bohr radius of the $\mu^-$-mesic atom.

(ii) If in the capture process (7.24) the $\mu^-$ is completely polarized, the angular distribution of the neutron is of the form

$$1 + a \cos \theta_1 ,$$  \hspace{1cm} (7.28)

where

$$\theta_1 = \angle (\sigma_{\mu} \mathbf{p}_n)$$  \hspace{1cm} (7.29)

representing the angle between the spin of the $\mu^-$-meson and the momentum of the neutron. The asymmetry parameter $a$ is given by

\[ a_\xi = -|S_s + S_v|^2 + |S_A + S_r|^2. \]  \hspace{1cm} (7.30)

(iii) The transition probability \((1/\tau)_{\text{rad}}\) for the radiative capture process of \(\mu^-\) in hydrogen,

\[ \mu^- + p \rightarrow n + \nu + \gamma, \]  \hspace{1cm} (7.31)

is

\[ (1/\tau)_{\text{rad}} = \frac{\epsilon^2 \eta}{6 \pi m c \xi} (1/\tau)_{\text{cap}} \]  \hspace{1cm} (7.32)

where

\[ \eta = |S_s|^2 + |S_v|^2 + 3|S_A|^2 + 3|S_r|^2. \]  \hspace{1cm} (7.33)

(iv) In process (7.31) the \(\gamma\)-ray is circularly polarized. The polarization parameter \(\beta\) may be defined as

\[ \beta = \frac{N_R - N_L}{N_R + N_L} \]  \hspace{1cm} (7.34)

where \(N_R\) and \(N_L\) are respectively the number of right-handed and left-handed \(\gamma\)-rays. The parameter \(\beta\) is given by

\[ \beta \eta = |S_s|^2 - |S_v|^2 - 3|S_A|^2 + 3|S_r|^2. \]  \hspace{1cm} (7.35)

Equation (7.35) is independent of either the state of polarization of the captured \(\mu^-\)-meson or the energy of the \(\gamma\)-rays.

(v) For the radiative capture of a 100\% polarized \(\mu^-\) the angular distribution of the \(\gamma\)-ray is of the form

\[ 1 + \beta \cos \theta_2 \]  \hspace{1cm} (7.36)

where \(\beta\) is given by Equation (7.34) and

\[ \theta_2 = \angle (\sigma_{\mu}, p_\gamma) \]  \hspace{1cm} (7.37)

where \(p_\gamma\) is the momentum of the \(\gamma\)-ray.

(vi) In the radiative capture process (7.31) the angular distribution of the \(\gamma\)-ray with respect to the momentum of the neutrino \(p_\nu\) is

\[ 1 + \gamma \cos \theta_3 \]  \hspace{1cm} (7.38)

where

\[ \theta_3 = \angle (p_\nu, p_\gamma) \]  \hspace{1cm} and

\[ \gamma \eta = -|S_s|^2 + |S_v|^2 - |S_A|^2 + |S_r|^2. \]  \hspace{1cm} (7.39)

In all the above expressions we neglect \(v/c\) terms for the nucleon wave function and we replace \(\psi_\mu\) by its value at the origin.

(vii) In (7.24) we assume that the law of conservation of leptons is valid. Otherwise instead of (7.24) and (7.31) we may have

\[ \mu^- + p \rightarrow n + \nu \]  \hspace{1cm} (7.40)

and

\[ \mu^- + p \rightarrow n + \bar{\nu} + \gamma. \]  \hspace{1cm} (7.41)
In the measurements of \((1/\tau)_{ca}\), \((1/\tau)_{rad}\), \(\theta_1\), \(\theta_2\), and \(\theta_3\) for reactions (7.40) and (7.41) the corresponding parameters \(\xi, \eta, \alpha, \beta, \gamma\) are replaced by \(\xi', \eta', \alpha', \beta', \gamma'\) where

\[
\xi' = \xi, \quad \eta' = \eta, \quad \alpha' = -\alpha, \quad \beta' = -\beta, \quad \text{and} \quad \gamma' = -\gamma. \quad (7.42)
\]

(viii) If the \(\mu^-\) capture process is invariant under time reversal then all \(G_i\) are real. The measurement of \(\alpha, \beta, \gamma, \eta, \xi\) affords a complete determination of the four coupling constants \(G_v, G_s, G_A, G_T\) plus a check on the validity of the conservation law of leptons. (It may also give a test on time reversal invariance.)

The above considerations can be extended to \(\mu^-\) capture in heavy nuclei. However, the uncertainty in the nuclear matrix elements would make the corresponding formulas less definite.
VIII. TIME REVERSAL INVARIANCE AND MACH'S PRINCIPLE

From various recent measurements we know that $P$ as well as $C$ are not conserved, at least in some of the weak interactions. There remains unanswered a very fundamental question which is whether $C\cdot P$ is invariant or not. Or, by the CPT theorem we may ask whether $T$ (time reversal) is invariant or not. If it turns out that $T$ may or may not be conserved, what will then be the implication? This leads us naturally to a discussion of Mach's principle.

Should we follow the spirit of Mach's principle, we would believe that the laws of physics cannot depend on the geometrical coordinate system that we happen to choose. There should exist no absolute system. The present asymmetry may then be made compatible with this interpretation of Mach's principle in two ways.

(i) If $T$ is invariant, then $C\cdot P$ is invariant. The right-left symmetry in space is retained by changing particle to antiparticle as we change from a right-handed system to a left-handed system.\textsuperscript{55}

(ii) If, experimentally, the weak interactions are found to be not invariant under $T$, the over-all symmetry may still be maintained by conjecturing the existence of two different kinds of elementary particles with the same masses, charges, and spins, but exhibiting opposite asymmetries under a space inversion. In such a picture, the observed right-left asymmetry is ascribed not to a basic non-invariance under space inversion but to a cosmologically local preponderance of one kind of the elementary particles over the other kind. Consequently, in this broader sense $P$ is still conserved. By the CPT theorem, all interactions are also invariant under $C\cdot T$. Thus, the time reversal symmetry can also be retained by changing particles to antiparticles as we reverse the chronological order of any sequence of events.

If this is the case, then there must exist two types of protons, $p_R$ and $p_L$, the right-handed one and the left-handed one. At the present time, the protons in the laboratory must be predominantly of only one kind, say $p_R$, which accounts for the observed asymmetry and the observed Fermi-Dirac statistical characters of the protons. This means that the free oscillation period between $p_R$ and $p_L$ must be longer than the age of the universe. They could, therefore, both be regarded as stable particles. It is reasonable to assume that there exists only one kind of electromagnetic field. Thus, in an experiment on pair productions by $\gamma$ radiation we expect the same cross sections for

$$\gamma \rightarrow p_R + \bar{p}_R$$

and for

\[ \gamma \rightarrow p_L + \bar{p}_L . \]  \hspace{1cm} (8.2)

The \( \bar{p}_L \) would appear to be a stable negative particle with a mass equal to that of a proton. The detection of such particles, if produced, may not be difficult.
Decay Modes of a ($\theta+\bar{\theta}$) System*

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(Received August 12, 1958)

Relations among the different states of a system consisting of a $\theta$ meson, a $\bar{\theta}$ meson, and a certain number of pions are discussed. Theorems concerning certain correlations between the production probabilities for charged and uncharged $\theta$ mesons and the decay modes ($\theta^0$ and $\bar{\theta}^0$) are obtained.

I

The purpose of this note is to study the decay of a system which consists of a $\theta$ meson, a $\bar{\theta}$ meson, and a certain number of pions. Such a system can be produced, e.g., in a collision between an antinucleon and a nucleon. We shall show that by the combined use of the isotopic-spin rotation operator and the charge conjugation operator there exist some interesting correlations, not only in production but also between some of the decay modes of the $\theta$ and $\bar{\theta}$. To be specific, let us first consider the reaction

$$\bar{p}+n \rightarrow \theta+\bar{\theta}+m\pi.$$  \hspace{1cm} (1)

We shall denote the final state of the $m$ pions by $D_m$ which represents both a specific charge distribution and a fixed momentum distribution of the pions. The charges and the decay modes of the $\theta$ and $\bar{\theta}$ mesons are of interest here. Let us define $P_{ij}(D_m)$ to be the probability of observing the $m$ pions with a distribution $D_m$ together with a $\theta_i$ of momentum $k_i$ and a $\bar{\theta}_j$ of momentum $k_j$ where $i$ $(j)$ runs over $+, -, 1, 2$ representing the cases in which $\theta_i$, $(\bar{\theta}_j)$ is $\theta^0$, $\theta^-$, $\theta^+$, or $\bar{\theta}^0$.

We assume that the two decay modes 1 and 2 are given by

$$\theta_i = \frac{1}{\sqrt{2}} (\theta^0 + \theta^0), \quad \bar{\theta}_j = \frac{-i}{\sqrt{2}} (\theta^0 - \theta^0);$$

i.e.,

$$\theta = \theta_i + i\bar{\theta}_j, \quad \bar{\theta} = \frac{1}{\sqrt{2}} (\theta_1 + i\theta_2).$$ \hspace{1cm} (2)

It is well known that this decomposition corresponds to the two experimental decay modes $\theta_1$ and $\theta_2$ if time-reversal invariance holds in the decay.

II

To study the dependence of $P_{ij}$ on $i$ and $j$, we consider all the states of a pair of $\theta$ and $\bar{\theta}$ of given momenta $k_i$ and $k_j$. There are eight possible states altogether.

Charge = 1: $(+\bar{0}), (\bar{0}+);$ \hspace{1cm} (3)

Charge = 0: $(+\bar{0}), (\bar{0}+), (0\bar{0}), (0\bar{0}).$

Here we adopt the obvious notation, e.g., $(+\bar{0})$ means the state with a $\theta^0$ meson having momentum $k_i$ and a $\bar{\theta}^0$ meson with momentum $k_j$. For the states with charge $= +1$, we have, by (2),

$$\begin{align*}
\frac{1}{\sqrt{2}} (+\bar{0}) &= \frac{1}{\sqrt{2}} (+1) - \frac{1}{\sqrt{2}} (+2), \\
\frac{1}{\sqrt{2}} (0\bar{0}) &= \frac{1}{\sqrt{2}} (-1) + \frac{1}{\sqrt{2}} (-2).
\end{align*}$$ \hspace{1cm} (4)

A general state of charge $+1$ is a superposition of these two states. One sees that in any superposition the decay modes $(+1)$ and $(+2)$ always have the same probability. Also the modes $(1+) + (2+)$ always have the same probability. Similar considerations can be extended to the states with charge $= 0$ and $-1$. One easily proves in this way the following theorem:

Theorem 1: \hspace{1cm} (5)

$$P_{+1}(D_m) = P_{+1}(D_m),$$ \hspace{1cm} (5)

$$P_{+2}(D_m) = P_{+2}(D_m),$$ \hspace{1cm} (6)

$$P_{-1}(D_m) = P_{-2}(D_m),$$ \hspace{1cm} (7)

$$P_{-2}(D_m) = P_{-1}(D_m).$$ \hspace{1cm} (8)

In proving this theorem the only assumptions are that the decomposition (2) holds and that the total strangeness of the pair $\theta\bar{\theta}$ is zero (so that, e.g., the pair $\theta^+\bar{\theta}^0$ is excluded).

III

It is possible to obtain more identities if one uses the conservation of isotopic spin and of the charge conjugation operator in the strong interactions. To do this, one has to study the transformation of the eight states (3) under an isotopic spin rotation and/or charge conjugation. The problem is identical to that of the transformation of the states describing a nucleon-antinucleon

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### Table I. Eigenfunctions for which strangeness $S=0$. Notations are explained in the text.

<table>
<thead>
<tr>
<th>$Q=I_1$</th>
<th>$I$</th>
<th>$G$</th>
<th>Form a</th>
<th>Wave function</th>
<th>Form b</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>−1</td>
<td>$2\psi^*[(+0)+(-0)]$</td>
<td>$=\psi[+(+1)+(-2)+(+1)-(-2)]$</td>
<td>$=\psi[+(+1)+(-2)+(+1)+(-2)]$</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>$2\psi^*[(+0)-(0+)]$</td>
<td>$=\psi[+(+1)+(-2)+(+1)+(-2)]$</td>
<td>$=\psi[+(+1)+(-2)+(+1)+(-2)]$</td>
</tr>
<tr>
<td>−1</td>
<td>1</td>
<td>−1</td>
<td>$-2\psi^*[(0+)-(-0-)]$</td>
<td>$=\psi[+(+1)+(+2)+(+1)+(-2)]$</td>
<td>$=\psi[+(+1)+(+2)+(+1)+(-2)]$</td>
</tr>
<tr>
<td>−1</td>
<td>1</td>
<td>1</td>
<td>$-2\psi^*[(0-)-(0+)]$</td>
<td>$=\psi[+(+1)+(+2)+(+1)+(-2)]$</td>
<td>$=\psi[+(+1)+(+2)+(+1)+(-2)]$</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>−1</td>
<td>$\psi[+(-++)(00)+(00)-(+++)]$</td>
<td>$=\psi[+(+)+(-++)+(11)+(22)]$</td>
<td>$=\psi[+(+)+(-++)+(11)+(22)]$</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>$\psi[+(-++)(00)-(00)+(+++)]$</td>
<td>$=\psi[+(+)+(-++)-(11)+(22)]$</td>
<td>$=\psi[+(+)+(-++)-(11)+(22)]$</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>−1</td>
<td>$\psi[+(+-)(00)-(00)+(+++)]$</td>
<td>$=\psi[+(+-)(00)-(00)+(+++)]$</td>
<td>$=\psi[+(+-)(00)-(00)+(+++)]$</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>$\psi[+(+-)(00)-(00)+(+++)]$</td>
<td>$=\psi[+(+-)(00)-(00)+(+++)]$</td>
<td>$=\psi[+(+-)(00)-(00)+(+++)]$</td>
</tr>
</tbody>
</table>

Following the arguments used in reference 4 for studying the latter problem, we first consider the four states of a single $\theta$ or a single $\bar{\theta}$:

$$
\begin{bmatrix}
\psi^+

\psi^0

\psi^0

-\psi^-
\end{bmatrix}
$$

(9)

The isotopic spin operators which operate on these states are

$$
I_1=\frac{1}{2}
\begin{bmatrix}
0 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
, \quad I_2=\frac{1}{2}
\begin{bmatrix}
0 & -i & 0 & 0 \\
i & 0 & 0 & 0 \\
0 & 0 & 0 & -i \\
0 & 0 & i & 0
\end{bmatrix}
, \quad I_3=
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & -1
\end{bmatrix}
$$

(10)

Instead of discussing the charge conjugation operator $C$, we define, as in reference 4,

$$
G=C \exp[i\pi I_3]
$$

(11)

and obtain

$$
G=
\begin{bmatrix}
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
-1 & 0 & 0 & 0 \\
0 & -1 & 0 & 0
\end{bmatrix}
$$

(12)

The operator $G$ commutes with $I$. The strangeness number operator is diagonal in this representation,

$$
S=
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & -1 & 0 \\
0 & 0 & 0 & -1
\end{bmatrix}
$$

(13)

We notice that

$$
SG+GS=0
$$

(14)

For a system of two particles with momenta $k_1$ and $k_2$, each described by a four-component wave function (9), the operators $I$ and $S$ are additive, whereas $G$ is multiplicative. Of the sixteen states of the two-particle system, eight belong to $S=0$ and are given in (3). For these states, the charge

$$
Q=I_3
$$

(15)

and $P, G, S$ all commute and can be simultaneously diagonalized. The eight eigenstates can be easily constructed and are displayed in Table I in the column headed by “Form a.” In the next column “Form b,” these eigenfunctions are rewritten by the use of (2) so that their decay modes are explicitly exhibited.

In reaction (1) the process in general goes through many channels of $I$ and $G$ which produce interference effects. However, the interference between different $G$ values disappears if we average over the collisions $(\bar{p}n)$ and $(n\bar{p})$, where the states are defined in a manner similar to Eq. (3). To see this, one writes

$$
(\bar{p}n)=\frac{1}{2}[\bar{p}n]+[\bar{p}n]-[\bar{p}n]-[\bar{p}n]
$$

$$
(n\bar{p})=\frac{1}{2}[n\bar{p}]+[n\bar{p}]-[n\bar{p}]-[n\bar{p}]
$$

The term $(\bar{p}n)+[n\bar{p}]$ belongs to $G=-1$ and the term $(\bar{p}n)-[n\bar{p}]$ to $G=+1$. The interference terms between $G=\pm 1$ therefore have opposite signs in $(\bar{p}n)$ and $(n\bar{p})$ and cancel exactly upon taking the average. In other words, after the average one may consider (1) as going through incoherent interfering channels with definite $G$ values. Since the pions are eigenstates of $G$, for any given distribution $D_n$ of the pions, there are therefore no interferences between the states of the $(\bar{p}n)$ system with different $G$ values. Table I shows then that for charge $Q=1$, the first two rows cannot interfere and one obtains, in addition to (5) and (6), the identity

$$
P_{+1}(D_n)=P_{+1}(D_n)
$$

These considerations are easily extended to states with $Q=0$ and $-1$.

### Theorem 2
Consider $\beta+n$ and $n+\bar{p}$ which are related by the substitution of a $\beta$ by an $n$ with equal momentum and spin and vice versa. After summation over these two initial states, the partial cross sections
satisfy
\[ P_{\Delta 1}(D_m) = P_{\Delta 2}(D_m) = P_{\Delta 3}(D_m) = P_{\Delta 4}(D_m), \]
\[ P_{11}(D_m) = P_{21}(D_m), \]
\[ P_{12}(D_m) = P_{22}(D_m), \]
and
\[ P_{11}(D_m) + P_{21}(D_m) + P_{22}(D_m) + P_{23}(D_m) = P_{+}(D_m) + P_{-}(D_m). \]

We thus have the interesting result that in those collisions where \( \theta^o \) and \( \theta^q \) are produced, their decay modes \( (\theta^o \pm \theta^q) \) are in general not independent of each other but are related in a manner given by Eqs. (17)–(19).

In the capture of an antiproton by a neutron, both at rest, it is clear that theorem 2 applies.

A simple consequence of (16)–(19) is that in a collision \( \bar{p} + n \), one has
\[ \frac{1}{2}(n_+ + n_-) = n_1 = n_2, \]
where \( n \) is the total number of \( \theta_i \) produced \( (i = 1, 2, +, -) \) integrated over all angles.

\[ \bar{n} + \rho \rightarrow \theta + \bar{\theta} + m \pi. \]

IV

It is easy to see that the same identities (theorems 1 and 2) are also valid for the reaction
\[ \bar{n} + n \rightarrow \theta + \bar{\theta} + m \pi. \]

Furthermore, for both reactions (1) and (1') additional equalities and inequalities may be obtained when \( m \) takes on some specific small values. In all cases, we found it convenient to use the eigenstates tabulated in Table I. They are useful also in discussing
\[ \rho + \bar{p} \rightarrow \theta + \bar{\theta} + m \pi, \]
and
\[ n + \bar{n} \rightarrow \theta + \bar{\theta} + m \pi. \]
Some Considerations on Global Symmetry

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(Received February 3, 1961)

If the recently discovered $Y^*$ state is related to the $T = \frac{1}{2}, J = \frac{3}{2}$ resonance in $\pi^0\pi^-$ scattering, global symmetry considerations should become relevant. In this paper, global symmetry is discussed with a view to understanding its group structure. Also discussed is a possibility of reconciling the conflict, pointed out by Pais, between certain experimental results and global symmetry. The partial widths of the $Y^*$ state are calculated and also those of the companion excited states $Z^*$ and $Z''$. A generalization of the quantum number $G$ is discussed.

1. INTRODUCTION

Recent experiments have established the existence of an excited state $Y^{*\pm}$ in the $\Delta^\pm\pi^0$ system. The spin and parity of the state are not yet measured. As discussed in reference 1, the state shows certain resemblance to the $J = \frac{3}{2}, T = \frac{1}{2}$, $\rho$ state resonance $N^\prime$ of the $\rho \pm \pi$ system, and the resemblance is reminiscent of the concept of global symmetry.

In this paper we proceed along this line of thinking and assume that indeed the $Y^{*\pm} = \Delta^\pm\pi^0$ resonance is in the $J = \frac{3}{2}, \rho$ state, and that the resonance is related to the $J = \frac{3}{2}, T = \frac{1}{2}$, $\rho$-state resonance $N^\prime$ of the $\rho \pm \pi$ system by global symmetry. To analyze this relation it is necessary to know the quantum numbers of various states with respect to the global symmetry operations. It is therefore important to know the structure of the global symmetry group. Now global symmetry means some symmetry, larger than isotopic spin invariance, that describes an approximate analogy between the various baryons. But in the literature its group property has not been fully discussed. We shall in this paper formulate in mathematical terms the requirements that global symmetry must satisfy. It appears that the simplest group $G$ satisfying these requirements can be generated by three independent 2-dimensional unitary unimodular transformations together with a discrete transformation. Adopting this group as the global symmetry group we then try to assign quantum numbers to the various particles and the resonance states $Y^\pm, N^\prime$.

Certain approximate relations are then written down between the widths of $Y^\pm$ and $N^\prime$, and also for the various partial widths of $Y^\pm$. Companion resonance states $Z^\pm$ and $Z^{\prime\prime}$ are also discussed.

It has been pointed out by Pais\(^a\) that any kind of global symmetry is in conflict with certain experimental facts. We suggest in Sec. 6 that if global symmetry is needed to understand the resonant state $Y^\pm$, a way to resolve Pais' conflict is to have the $K$ mesons as a mixture of states which have different quantum numbers under the global symmetry transformations. The interactions between each of these states and other particles could still predominantly satisfy global symmetry. Such a picture, while not completely satisfactory, does offer a possible consistent scheme incorporating global symmetry that leads to useful experimental information.

It should be emphasized that much of our results about widths have already been discussed in the literature from the viewpoint of symmetry considerations.\(^b,c\) Furthermore a detailed calculation using a specific dynamical model has been performed by Amati, Stanghellini, and Vitale\(^d\) for the states $Y^\pm$ and $Z^\pm$. Various discussions on the global symmetry group properties have also existed in the literature.\(^e,f\) The present paper is written not in the spirit of presenting something entirely new, nor even in that of presenting something which we believe to be necessarily relevant to physical facts. But if a similarity between $Y^\pm$ and $N^\prime$ exists, an analysis along the present line would be useful.

For completeness we include in Sec. 8 a discussion of charge conjugation invariance, together with a generalization of the quantum number $G$. Also included are some remarks in Sec. 9 concerning a global symmetry that does not put $Z$ and nucleons in the same multiplet.

2. REQUIREMENTS ON THE GLOBAL SYMMETRY GROUP

The global symmetry group must by definition contain the isotopic spin group and the strangeness group [defined by the operators $exp(is\theta)$ or $exp(i(S+N)\theta)$, where $S$ = strangeness and $N$ = baryon number; the strangeness group commutes with the isotopic spin group]. It has an $8 \times 8$ unitary representation to which the 8 baryons belong. [Cf. Sec. 9 for the case of a symmetry between $N$, $\Sigma$, and $\Lambda$ only.] In order that the

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8 baryons be analogous to each other under the global symmetry group, the representation must be irreducible. For the isotopic spin rotation subgroup the \(8 \times 8\) representation breaks up into two doublets \((N\) and \(Z)\), one triplet \((\Sigma)\), and one singlet \((\Lambda)\). For the strangeness subgroup this breakup must conform with the usual assignments of \(S+\overline{N} = +1, -1, 0\) for \(N, \Sigma, \Xi, \) and \(\Lambda\), respectively.

To state the above requirements explicitly we introduce as usual the states

\[
\begin{align*}
Y^+ &= \Sigma^+, \\
Y^0 &= \frac{1}{\sqrt{2}} (\Sigma^0 + \Lambda^0), \\
Z^0 &= \frac{1}{\sqrt{2}} (\Sigma^0 - \Lambda^0), \\
Z^- &= \Sigma^-, \\
\end{align*}
\]

and write

\[
\begin{align*}
N &= \begin{pmatrix} \rho^+ \\ n \end{pmatrix}, \\
Y &= \begin{pmatrix} Y^+ \\ Y^0 \end{pmatrix}, \\
Z &= \begin{pmatrix} Z^0 \\ Z^- \end{pmatrix}, \\
\Xi &= \begin{pmatrix} \Sigma^+ \\ \overline{\Xi} \end{pmatrix}. \\
\end{align*}
\]

It is now useful to introduce (operating on the column matrix \(B\)) the following three sets of operators, each set satisfying the commutation relations for angular momenta:

\[
\begin{bmatrix}
\sigma_1 & 0 & 0 & 0 \\
0 & \sigma_1 & 0 & 0 \\
0 & 0 & \sigma_1 & 0 \\
0 & 0 & 0 & \sigma_1
\end{bmatrix}, \quad (i = 1, 2, 3), \\
\begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}, \quad \mathfrak{M}_2 \equiv \frac{1}{2} I \\
\begin{bmatrix}
0 & I & 0 & 0 \\
I & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}, \quad \mathfrak{M}_3 \equiv \frac{1}{2} I
\]

where

\[
\begin{align*}
\sigma_1 &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, & \sigma_2 &= \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, & \sigma_3 &= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \\
I &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.
\end{align*}
\]

Clearly \(\mathcal{L}_i, \mathfrak{M}_j, \mathfrak{M}_k\) all commute for any \(i, j, k\).

The physical observables, \(Q = \) charge, \(S = \) strangeness, \(N = \) baryon number, \(T_1, T_2, T_3 = \) isotopic spin are related to \(\mathcal{L}, \mathfrak{M}_j, \) and \(\mathfrak{M}_k\) by

\[
\begin{align*}
T_1 &= \mathcal{L}_z + 3\mathfrak{M}_z, \\
\frac{1}{2}(S+N) &= \mathfrak{M}_3, \\
Q &= T_1 + \mathfrak{M}_3 = \mathcal{L}_z + 3\mathfrak{M}_z + \mathfrak{M}_3.
\end{align*}
\]

The \(8 \times 8\) representation must (i) contain \(\exp(i T_1 \mathfrak{M}_3)\), \(\exp(i T_2 \mathfrak{M}_3)\), \(\exp(i T_3 \mathfrak{M}_3)\), \(\exp(i \mathfrak{M}_j \mathfrak{M}_k)\). In other words, \(T_1, T_2, T_3, \mathfrak{M}_3\) are among the infinitesimal generators of the representation. Furthermore (ii) the \(8 \times 8\) representation is irreducible.

Many possible \(8 \times 8\) representations of groups can be found that satisfy (i) and (ii). Since we are not interested in unnecessary additional symmetries, we take the group to be isomorphic with the representation, and shall identify a group element with its \(8 \times 8\) representation. The simplest such possibility, \(G_8\), is discussed in the following sections. Other possibilities are discussed in Appendix A.

### 3. The Group \(G_8\)

The group contains arbitrary \(\mathcal{L}\) transformations [i.e., 
\(U_\mathcal{L} = \exp(i \mathcal{L}_1 T_1 + \mathcal{L}_2 T_2 + \mathcal{L}_3 T_3)\)], where \(T_1, T_2, T_3\) are real numbers], arbitrary \(\mathfrak{M}_j\) transformations [\(\exp(i \mathfrak{M}_j \mathfrak{M}_3 + m \mathfrak{M}_3)\)], which mixes \(Y\) and \(Z\), and arbitrary \(\mathfrak{M}_k\) transformations [\(\exp(i \mathfrak{M}_k \mathfrak{M}_3 + n \mathfrak{M}_3 + \mathfrak{M}_3)\)], which mixes \(N\) and \(Z\)]. The product of all these is reducible since no mixing of \(N, Z\) with \(Y, Z\) has been introduced. To effect such a mixing we introduce the discrete element\(^{10}\)

\[
R = \begin{pmatrix}
0 & 0 & I & 0 \\
0 & 0 & 0 & 0 \\
I & 0 & 0 & 0 \\
0 & I & 0 & 0
\end{pmatrix}
\]

\(^{10}\) This discrete operator has been discussed by various authors. See reference 8.
and other necessary elements to form a group. The elements of the group are then the $8\times8$ matrices of the form

\[
\begin{pmatrix}
a I & b I \\
-b^* a I & a^* I
\end{pmatrix}
\]

and those of the form

\[
\begin{pmatrix}
0 & a I \\
-b^* a^* I & 0
\end{pmatrix}
\]

Here

\[
\begin{pmatrix}
a & b \\
-b^* a^*
\end{pmatrix} = U = \text{arbitrary } 2\times2 \text{ unimodular unitary matrix},
\]

and

\[
\begin{pmatrix}
a' & b' \\
-b'^* a'^*
\end{pmatrix} = U' = \text{arbitrary } 2\times2 \text{ unimodular unitary matrix},
\]

This group will be called $G_6$. It has an invariant subgroup (11) which is the direct product of three $SU_2$, and the quotient of the group by this invariant subgroup is the two-element group. [It is, however, not the direct product of the two-element group with the invariant subgroup.]

In Appendix A we shall give a few other possible groups satisfying conditions (i) and (ii).

---

**4. IRREDUCIBLE REPRESENTATION OF $G_6$**

For any representation of $G_6$, the infinitesimal operators $E_3, E_3, E_5, E_6, E_8, E_9, E_{10}, E_{11}, E_{12}$ form three sets of commuting angular momenta. One can diagonalize $E_3, E_3, E_5, E_6, E_9, E_{10}, E_{11}, E_{12}$ simultaneously. Now

\[
R E R^{-1} = E, \\
R E_{12} R^{-1} = E_{12},
\]

Hence in any representation the set of eigenvalues of $E_3$ must be the same as those of $E_3$. One has therefore two kinds of irreducible representations:

(a) $E_3, E_5, E_6$ have unique eigenvalues $\mathcal{E}(\pm 1)$, $\mathcal{E}(\pm 1)$, $\mathcal{E}(\pm 1)$, respectively. Since $R$ commutes with $E_3, E_5, E_6$, the state with $\mathcal{E}_3 = \mathcal{E}_5 = \mathcal{E}_6$ is an eigenstate of $R$. The eigenvalue $\lambda$ can be $\pm 1$. We denote this representation by the symbol $(\mathcal{E}_3 E_5 E_6)$, where $2\mathcal{E} = \text{integer} \geq 0$, $2 \mathcal{E}_5 = \text{integer} \geq 0$, $\lambda = \pm 1$.

The states of this representation are designated by $E_3, E_5, E_6$, each running in integral steps between and including $\pm \mathcal{E}_3, \pm \mathcal{E}_5, \pm \mathcal{E}_6$ respectively. The operator $R$ switches the indices $E_3, E_5, E_6$ for a state:

\[
R [E_3 = a, E_5 = b] = \lambda [E_3 = b, E_6 = a].
\]

(b) $E_3$ has a unique eigenvalue $\mathcal{E}(\pm 1)$. $E_5$ and $E_6$ each has two eigenvalues $\mathcal{E}(\pm 1)$ and $\mathcal{E}(\pm 1)$ where $\mathcal{E}_5 \neq \mathcal{E}_6$. We denote this representation by the symbol $(\mathcal{E}_3 E_5 E_6)$, where $2\mathcal{E}_3, 2\mathcal{E}_5$, and $2\mathcal{E}_6$ are integers $\geq 0$ and $2\mathcal{E}_3 > 2\mathcal{E}_5$.

The states of this representation are states for which

\[
\mathcal{E}_3 = \mathcal{E}(\pm 1), \mathcal{E}_5 = \mathcal{E}(\pm 1), \mathcal{E}_6 = \mathcal{E}(\pm 1), \mathcal{E}_7 = \mathcal{E}(\pm 1), \mathcal{E}_8 = \mathcal{E}(\pm 1), \mathcal{E}_9 = \mathcal{E}(\pm 1), \mathcal{E}_{10} = \mathcal{E}(\pm 1); \]

and

\[
\mathcal{E}_3 = \mathcal{E}(\pm 1), \mathcal{E}_5 = \mathcal{E}(\pm 1), \mathcal{E}_6 = \mathcal{E}(\pm 1), \mathcal{E}_7 = \mathcal{E}(\pm 1), \mathcal{E}_8 = \mathcal{E}(\pm 1), \mathcal{E}_9 = \mathcal{E}(\pm 1), \mathcal{E}_{10} = \mathcal{E}(\pm 1), \mathcal{E}_{11} = \mathcal{E}(\pm 1), \mathcal{E}_{12} = \mathcal{E}(\pm 1); \]

The operator $R$ switches the states between these two sets. In a suitable representation, $R$ satisfies

\[
R [\mathcal{E}_3 (E_5 E_6) = \mathcal{E}_3 (E_5 E_6)].
\]

The dimensions of the irreducible representations are tabulated in Table I. Also given are the characters of the representations. From the characters the decomposition of the direct product of two representations can be easily
found in the standard way. (Except for the quantum number \(\lambda\), it can also be found by the usual vector sum rule for \(L\), \(S\), and \(\Sigma\) separately.)

5. QUANTUM NUMBERS

To assign quantum numbers to the states we first notice that Eqs. (7)–(9) give the isospin, the strangeness, and the charge in terms of these quantum numbers.

The 8 baryons clearly belong to the representation \((1,1,0)\). It seems natural that the pions should be assigned to the 3\(\times\)3 representation \((1,0,0,0)\). The two possibilities \(\lambda_\pi = \pm 1\) are, of course, physically different, and differentiable. It seems natural to assign the \(K\) mesons to the representation \((0,1,\frac{1}{2},\lambda_K)\) with again the two possibilities \(\lambda_K = \pm 1\). These assignments are tabulated in Table II.

For the state \(N^* = \pi + p\) we notice that \(\pi + p\) always belongs to either \((1,1,0)\) \(T = \frac{3}{2}\), or \((1,\frac{1}{2},\frac{1}{2})\) \(T = \frac{1}{2}\). But \(N^*\) has a total \(T = \frac{1}{2}\). Hence it belongs to \((1,\frac{1}{2},\frac{1}{2})\). The natural assumption is therefore that \(Y^*\) is in the same multiplet structure \((1,\frac{1}{2},0)\), as indicated in Table II. Since \(Y^*\) can go into \(\Lambda + \pi\), its isospin \(T = 1\). The multiplet \((1,\frac{1}{2},0)\) also contains a \(T = 2\) state \((\frac{3}{2},0)\) which will be called \(Z^*\). In addition to the 3 \(Y^*\) states and 5 \(Z^*\) states there should also be 4 \(\Sigma^*\) states with \(T = \frac{3}{2}\).

6. BREAKDOWN OF GLOBAL SYMMETRY

Even if global symmetry has any valid basis, there must be relatively strong interactions that violate it. One manifestation of this violation lies in the mass difference between the hyperons. Another manifestation was first pointed out by Pais, who showed that the following reactions

\[ \pi^+ + p \rightarrow \Sigma^+ + K^+, \]
\[ K^+ + n \rightarrow \Sigma^0 + p, \]

and many others violate global symmetry. For the group \(G_0\) discussed above, the conservation of \(L_3\) is violated by both (16) and (17).

In face of these difficulties, does global symmetry have any validity at all? And if it has, does it ever produce useful physical information?

It would be difficult to answer these questions. But if the answers to the above questions are affirmative, presumably the baryons and the states \(N^*, Y^*\) allow more directly the application of global symmetry than reactions (16) and (17). For example, if the global-symmetry-breaking interactions produce relatively little mixing for the baryons, pions and \(Y^*, N^*\), but produce large mixing for the \(K\) mesons, then apparent violation of global symmetry is not unnatural for (16) and (17). While the mixing may be the multiplet \((0,1,\frac{1}{2})\) with any multiplet possessing a \(T = \frac{1}{2}\), \(S = \frac{1}{2}\) component, it seems that the mixing of \((0,1,\frac{1}{2})\) with \((1,\frac{1}{2},\frac{1}{2})\) is the simplest possibility.

In this view, then, the usual interactions (baryons, pions, and \(K\) mesons) are regarded as predominantly globally symmetrical. The globally unsymmetrical interactions give rise to, among others, two effects: (a) mass splitting of the states within each multiplet. (b) a strong mixing for the \(K\) meson of a \((1,\frac{1}{2},\frac{1}{2})\) \(T = \frac{1}{2}\) component with the \((0,1,\frac{1}{2})\) state. The globally unsymmetrical interactions may, for example, have a very small range, so that the two effects (a) and (b) are the only ones that one need consider as causing global unsymmetry in the zeroth approximation. The influence of global unsymmetry is then quite limited in scope, though not in magnitude, and one can derive consequences that can be checked with experimental information. [This is true only insofar as one does not probe into the very small range where the strong unsymmetric force is assumed to be effective. It may be instructive to recall the well-known symmetry between \(e^\pm\) and \(\mu^\pm\).] In that case, the asymmetry between these two particles

<table>
<thead>
<tr>
<th>Particle</th>
<th>Representation</th>
<th>(L)</th>
<th>(L_3)</th>
<th>(S)</th>
<th>(S_\pi)</th>
<th>(\lambda)</th>
<th>(G_1)</th>
<th>(T)</th>
<th>(R)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(p, n)</td>
<td>((1,\frac{1}{2},0))</td>
<td>(\pm)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>(\frac{1}{2})</td>
<td>(\pm)</td>
<td>(\mp)</td>
<td></td>
</tr>
<tr>
<td>(\Sigma^+, \Sigma^0)</td>
<td>(\pm)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>(\Sigma^0)</td>
<td>(\pm)</td>
<td>(\mp)</td>
<td>1, 0</td>
<td></td>
</tr>
<tr>
<td>(\Sigma^-, \Sigma^-)</td>
<td>(\pm)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>(\Sigma^-)</td>
<td>(\pm)</td>
<td>(\mp)</td>
<td>1, 0</td>
<td></td>
</tr>
<tr>
<td>(\pi^+)</td>
<td>((0,1,\frac{1}{2},\lambda_K))</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>(\lambda_K)</td>
<td>0</td>
<td>(\frac{1}{2})</td>
<td></td>
</tr>
<tr>
<td>(\pi^0)</td>
<td>(\lambda_\pi = \pm 1)</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>(\lambda_\pi)</td>
<td>0</td>
<td>(\frac{1}{2})</td>
<td></td>
</tr>
<tr>
<td>(K^+)</td>
<td>(\lambda_K = \pm 1)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>(\lambda_K)</td>
<td>0</td>
<td>(\frac{1}{2})</td>
<td></td>
</tr>
<tr>
<td>(K^0)</td>
<td>(\lambda_K = \pm 1)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>(\lambda_K)</td>
<td>0</td>
<td>(\frac{1}{2})</td>
<td></td>
</tr>
<tr>
<td>(K^-)</td>
<td>(\lambda_K = \pm 1)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>(\lambda_K)</td>
<td>0</td>
<td>(\frac{1}{2})</td>
<td></td>
</tr>
<tr>
<td>(N^*)</td>
<td>((1,1,0))</td>
<td>(\ldots)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>(\ldots)</td>
<td>(\ldots)</td>
<td>(\frac{1}{2})</td>
<td></td>
</tr>
<tr>
<td>(Y^*)</td>
<td>((1,\frac{1}{2},\frac{1}{2}))</td>
<td>(\ldots)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>(\ldots)</td>
<td>(\ldots)</td>
<td>(\frac{1}{2})</td>
<td></td>
</tr>
</tbody>
</table>

Table II. Quantum number assignments for particles and excited states. \(T = L + S\) = isotopic spin, \(P = \mp\) = parity of \(K\) meson. The quantum number \(G_1\) is explained in Sec. 8. In this table only the \((0,1,\frac{1}{2},\lambda_K)\) part is listed for the \(K\) mesons.
TABLE III. Phase-space factor $\Omega$, projection weight $w$, and relative partial widths of resonance levels. The weights $w$ are calculated from the quantum number assignments. The phase space factor $\Omega$ is computed from experimental resonance energies for $N^*$ and $\Lambda^*$, and from an assumed energy spectrum for $Z^*$ and $\Sigma^*$. The partial widths of other disintegration processes, such as $Z^* \rightarrow \pi^+ + \pi^-$ etc., can be inferred from the table through a simple isotopic spin rotation.

<table>
<thead>
<tr>
<th>Particle</th>
<th>Total energy (MeV)</th>
<th>Disintegration products $\Omega$ (MeV)$^2$</th>
<th>Computed relative partial width $w$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(N^<em>)^</em>$</td>
<td>1237</td>
<td>$\pi^+ + \pi^- + \gamma$</td>
<td>$9.7 \times 10^6$</td>
</tr>
<tr>
<td>$Y^*$</td>
<td>1385</td>
<td>$\pi^+ + \pi^- + \gamma$</td>
<td>$7.3 \times 10^6$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\pi^+ + \pi^0 + \gamma$</td>
<td>$1.6 \times 10^6$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$2\pi^+ + \gamma$</td>
<td>$1.6 \times 10^6$</td>
</tr>
<tr>
<td>$Z^{++}$</td>
<td>$\sim 1539(%)$</td>
<td>$\pi^+ + \Sigma^+$</td>
<td>$18(%) \times 10^6$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\sim 1539(%)$</td>
<td>$\sim 1.9(%)$</td>
</tr>
<tr>
<td>$Z^{**}$</td>
<td>$\sim 1637(%)$</td>
<td>$\pi^+ + \Sigma^0$</td>
<td>$15(%) \times 10^6$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\sim 15(%)$</td>
<td>$\sim 1.5(%)$</td>
</tr>
</tbody>
</table>

seems to be completely characterized by their large mass difference which, presumably, is also generated by some unsymmetrical forces, strong in magnitude but remarkably limited in its symmetry destroying effects.]

If one asks how it happens that only the $K$ particles have a strong mixing, a possible answer could be that the globally unsymmetrical interaction of two multiplets $(1, \frac{1}{2})$ and $(0, \frac{1}{2})$ happen to lie relatively close together and, therefore, result in large mixings for the states with $T=\frac{3}{2}$. It would then be reasonable to expect the existence of other excited states $K^*$.

In such a picture the $K$ meson is a mixture of $(0, \frac{1}{2})$ and $(1, \frac{1}{2})$, each of which, in interacting with the other particles, still predominantly satisfies global symmetry. Thus, e.g., in reactions such as $K^- + \text{baryon}$ with multiple pion productions, one can apply the global symmetry arguments and obtain equalities and inequalities between the various related processes.

7. POSITIONS AND WIDTHS OF $N^*$, $Y^*$, $Z^*$, AND $\Sigma^*$

The discussion of Sec. 6 suggests a zeroth order calculation of the partial widths of $N^*Y^*Z^*$ and $\Sigma^*$, for the processes tabulated in Table III. These processes represent a transition from a $(\frac{1}{2}, \frac{1}{2}, 0)$ multiplet to a product of a $(1, \frac{1}{2}, 0)$ multiplet and a $(0, 0, \lambda)$ multiplet. The decomposition therefore yields unique weights $w$ which are related to the squares of the appropriate transition amplitude. The calculation of these weights from the usual tables of Clebsch-Gordan coefficients is straightforward and the result is tabulated in Table III. Besides these weights due to the projection of the initial state on final states, there is also a phase-space-potential-barrier factor $\Omega$ for the $\gamma$-wave state. We take it to be given by

$$\Omega = q^2 E_B/(E_B + E_\gamma),$$

where $q$ = momentum of pion in the rest system of the resonance state, $E_B$ = total energy of the final baryon, and $E_\gamma$ = total energy of the final pion. In the approximation that other effects due to global asymmetrical interactions are neglected, the partial widths of each resonance level are proportional to the appropriate products of $w$ and $\Omega$. To calculate $\Omega$ one needs the excitation energy of the resonance states. For $N^*$ and $Y^*$ we take the values in reference 1. To guess at the energies of $Z^*$ and $\Sigma^*$ we write the total energy $E$ of the excited state in the multiplet $(\frac{3}{2}, 0)$ in the form

$$E = E_B + E_\gamma$$

where $\alpha'$, $\beta'$, and $\gamma'$ are constants. Similarly, for the 8 baryons in the multiplet $(\frac{1}{2}, 0)$ we have an analogous expression

$$E = E_B + \alpha E_\gamma + \beta E_\gamma + \gamma (\Lambda - \frac{1}{2}) + \gamma' (\Sigma - \frac{1}{2}),$$

where $\alpha = E_B + \gamma \approx 77$ Mev,

$$\beta = \frac{1}{2} (E_B + E_\gamma) - \frac{1}{2} (3E_B + E_\gamma) \approx -59$$ Mev,

$$\gamma = -(E_B - E_\gamma) \approx -380$$ Mev.

One sees that by taking

$$\alpha' = \alpha, \beta' = \beta, \gamma' \approx -400$$ Mev,

one obtains the experimental resonance energy for $Y^*$. With this choice the resonance energies for $Z^*$ and $\Sigma^*$ can be computed and are tabulated in Table III, with the corresponding phase space factors $\Omega$.

The last column of Table III shows a smaller total width for $Y^*$ than $N^*$ [in the ratio of approximately 0.56:1], and shows a very small branching ratio of $Y^* \rightarrow \Sigma + \pi$. Both of these are in general agreement with experimental information.\(^1\)

8. CHARGE CONJUGATION INVARIANCE

With the inclusion of the unitary operator $C$, representing charge conjugation, the symmetry group is enlarged. The irreducible representations become larger in general, corresponding to, e.g., the fact that a particle and its antiparticle have the same mass. To study the combined group generated from $G_\frac{1}{3}$ and $C$ and the baryon number gauge transformation exp$(\lambda F)$, we start from their commutation relations. For the sake of clarity we shall formulate this discussion in theorems. We shall also only deal with particles and states that have transition matrix elements into $n$ baryons and antibaryons, $n=1, 2, 3, \ldots$.

Theorem 1.

$$G_t = C \exp[\imath (E_\gamma + 3n + 3\lambda)]$$

commutes with all elements of the group $G_\frac{1}{3}$.

Proof: For a single baryon-antibaryon the explicit representations of $C, \frac{1}{2}, 3n, 3\lambda, R$ are given in Appendix B. The theorem follows from a straightforward explicit

\(^1\)A similar guess on the masses of excited levels has been made by A. Pais (private communication). See also reference 7.
computation of the commutators. For other states the theorem follows because \( G_1, C, \exp(\pm i\mathbf{L}_2) \), \( \exp(\pm i\mathbf{L}_3) \), and \( \exp(\pm i\mathbf{L}_4) \) are all multiplicative for a collection of particles.

**Theorem 2.**

\[
G_1 N + NG_1 = 0, \quad [N, G_0] = 0, \quad G_1^2 = 1. \tag{24}
\]

**Proof:** This theorem follows directly from the explicit representation of Appendix B.

**Theorem 3.** The full group generated by \( G_0, G_1 \) and \( \exp(iN\theta) \) is the direct product group \( G_0 \times O_2 \), where \( O_2 \) is the group of all \( 2 \times 2 \) real orthogonal matrices with determinant \( \pm 1 \).

The proof of this theorem is again straightforward.

The irreducible representations of \( O_2 \) are either (a) \( 2 \times 2 \) in size, in which \( N = +\alpha \) and \( N = -\alpha \) \([\alpha = \text{integer}]\) each occurs once, representing physically a pair of particles, and \( G_1 \) switches the two states; or (b) the representation is of dimension \( 1 \times 1 \) in which \( N = 0 \) and either \( G_1 = +1 \) or \( G_1 = -1 \).

For a state with \( N = 0 \), the operator \( G_1 \) is one and the same numerical constant \((\pm 1)\) for all states in a multiplet of \( G_0 \). By (23), \( C \) brings one state into another in the same multiplet of \( G_0 \).

**Theorem 4.** For the pions, \( G_1 = -1 \).

**Proof:** The pions are eigenfunctions of \( \mathcal{H} \) with eigenvalue \( \mathcal{H} = 0 \). Hence \( G_1 = G_0 = -1 \).

**Theorem 5.** For the \( (0, \frac{1}{2}, \frac{1}{2}, \lambda) \), \( N = 0 \) representation, if the total angular momentum \( J = 0 \), and the system has transition matrix elements into a baryon-antibaryon pair, then

\[
G_1 \lambda = -1. \tag{25}
\]

For the \( (1, \frac{1}{2}, \frac{1}{2}, \lambda) \), \( N = 0 \) representation under the same assumption,

\[
G_1 \lambda = 1. \tag{26}
\]

**Proof:** In the notation of Appendix B, we have four possible states for the baryon-antibaryon pair that belong to \( (0, \frac{1}{2}, \frac{1}{2}) \), with \( \mathfrak{H}_3 = \frac{1}{2}, \mathfrak{H}_4 = \frac{1}{2} \):

\[
(\mathbf{Z}^0) + (\mathbf{Z}^1), \tag{27}
\]
\[
(\mathbf{Z}^0) + (\mathbf{Z}^1), \tag{28}
\]
\[
(\mathbf{Y}^0 + \mathbf{Z}^0) + (\mathbf{Y}^0 - \mathbf{Z}^0), \tag{29}
\]
\[
(\mathbf{Z}^0 + \mathbf{Y}^1) + (\mathbf{Z}^1 + \mathbf{Y}^0), \tag{30}
\]

where each curly bracket represents a state, for which the first symbol inside specifies the state of particle \( a \) and the second symbol that of particle \( b \). Consider a \([e \text{ number}] \) product wave function of an orbital part, a spin part, and a charge part \([\text{depending on the other quantum numbers, } \mathbf{L}_a, \mathbf{L}_b, \mathfrak{M}_a, \mathfrak{M}_b, \text{ etc.}]\) of the two particles. For a state \( J = 0 \), the product of the first two parts is antisymmetry in the interchange \( a \leftrightarrow b \). Hence the charge part is symmetric since the entire wave function must be antisymmetric. Now under \( a \leftrightarrow b \),

\[
(27) \leftrightarrow (28), \quad (29) \leftrightarrow (30). \tag{31}
\]

Hence the states are either

\[
(27) + (28) \text{ or } (29) + (30), \tag{32}
\]
or superpositions of these two. By using the explicit matrices listed in Appendix B it can be directly verified that under

\[
R: \quad (27) \leftrightarrow (29), \quad (28) \leftrightarrow (30),
\]
\[
G_1: \quad (27) \leftrightarrow - (30), \quad (28) \leftrightarrow - (29).
\]

Hence under \( RG_1 \) both wave functions in (31) remain themselves but change sign. Thus (25) is proved. A similar proof holds for (26).

Applied to the \( K \) mesons, if the \( K \) meson admixture \( (1, \frac{1}{2}, \frac{1}{2}) \) can be neglected, Theorem 5 states that

\[
G_1 = -\lambda K.
\]

While all the four states \( \mathfrak{M}_3 = \pm \frac{1}{2} \) and \( \mathfrak{M}_4 = \pm \frac{1}{2} \) are eigenstates of \( G_1 \) with eigenvalues \( -\lambda K \), only two: \( \mathfrak{M}_3 = \mathfrak{M}_4 = \pm \frac{1}{2} \) are eigenstates of \( R \) with eigenvalues \( \lambda K \).

If, furthermore, one assumes that time reversal invariance holds, then the two states \( K^0 \) and \( K^\pm \) have simple behaviors under \( R \). To see this, we notice that \( (G_1) \) is a numerical constant. Hence (23) shows that

\[
C |\mathfrak{M}_3 = \frac{1}{2}, \mathfrak{M}_4 = -\frac{1}{2} \rangle = -G_1 |\mathfrak{M}_3 = \frac{1}{2}, \mathfrak{M}_4 = \frac{1}{2} \rangle.
\]

If \( P \) is the parity operator, we have

\[
CP |\mathfrak{M}_3 = \frac{1}{2}, \mathfrak{M}_4 = -\frac{1}{2} \rangle = -G_1 P |\mathfrak{M}_3 = \frac{1}{2}, \mathfrak{M}_4 = \frac{1}{2} \rangle.
\]

Now \( K^0, K^\pm \) is an eigenstate of \( CP \) with eigenvalue \( +1 \), \((-1)\). Hence

\[
|K_1, \frac{1}{2}\rangle = |\mathfrak{M}_3 = \frac{1}{2}, \mathfrak{M}_4 = -\frac{1}{2} \rangle \mp (G_1 P) |\mathfrak{M}_3 = -\frac{1}{2}, \mathfrak{M}_4 = \frac{1}{2} \rangle.
\]

One obtains with the use of (15):

\[
R |K_1, \frac{1}{2}\rangle = \mp (G_1 P) |K_1, \frac{1}{2}\rangle = \pm (\lambda K P K) |K_1, \frac{1}{2}\rangle,
\]

where \( P K \) is the parity of \( K^0 \) with respect to, say, \( \lambda K \). Thus we obtain the entries in Table II for \( K^0, K^\pm \) under \( R \).

9. REMARKS

For a symmetry to exist between \( Y^* \) and \( N^* \) it is not necessary that all 8 baryons be brought into global symmetry. For example, one could have a symmetry between \( N, Y, \) and \( Z \) without \( \mathfrak{Z} \). A simple symmetry group in such a case\(^{11}\) is \( SU_2 \times SU_4 \) which has more parameters than \( G_0 \). The irreducible representations in such a case can be written down and an analysis like the above for \( G_0 \) can be made. Thus there would also be a companion \( Z^* \) \( T = 2 \) state together with \( N^* \) and \( Y^* \). The weight factors \( \omega \) for \( N^*, Y^*, Z^* \) remain the same as in Table III.

**APPENDIX A**

We give here several examples other than \( G_0 \) satisfying conditions (i) and (ii) of Sec. 2.

(A) The group \( G_1 \). The group is isomorphic with \( SU_2 \times SU_4 \) where \( SU_4 \) consists of the \( \mathbf{L} \) transformations

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\(^{11}\) T. D. Lee and C. N. Yang, Nuovo cimento 3, 749 (1956).
The infinitesimal generators of the invariant subgroup $	ext{SU}_2 \times 	ext{SU}_2 \times 	ext{SU}_2$ of $G_6$ are


g_1, g_2, g_3, (1+p_3)g_1, (1+p_3)g_3, (1+p_3)g_1g_3, (1-p_3)g_1, (1-p_3)g_3, (1-p_3)g_1g_3, (33)

or

\[ \sigma_i, \tau_i, p_3\tau_i. \]

The infinitesimal generator for (32) is

\[ p_3\sigma_1. \]

Taking the commutator of this generator with those listed in (33) one obtains additional generators. Altogether by repeatedly taking commutators one obtains the following 21 infinitesimal generators:

\[ \sigma_i, \tau_i, p_3\sigma_i, p_3\tau_i, p_3\sigma_i\tau_i. \]

Taking further commutators gives rise to no new independent generators. The group obtained from these 21 generators is $O_7$ which has a 2-1 homomorphisness with the group of 7x7 proper real orthogonal matrices $O_7$, as already discussed by various authors in the literature. To see this we define seven anticommuting Hermitian matrices

\[ \gamma_1 = \rho_1\sigma_1, \quad \gamma_2 = \rho_2\sigma_2, \quad \gamma_3 = \rho_3\sigma_3, \quad \gamma_4 = \rho_1\tau_1, \quad \gamma_5 = \rho_2\tau_2, \quad \gamma_6 = \rho_3\tau_3, \quad \gamma_7 = \rho_3\tau_1. \]

Then

\[ \gamma_\mu\gamma_\nu + \gamma_\nu\gamma_\mu = 2\delta_\mu\nu, \]

\[ \gamma_1\gamma_2\gamma_3\gamma_4\gamma_5\gamma_6\gamma_7 = -1. \]

The 21 infinitesimal generators (34) are then $i\gamma_\mu\gamma_\nu (\mu \neq \nu)$. The group generated by (34) is therefore of the form

\[ \exp(\sum a_\mu\gamma_\mu), \]

where $a_\mu$ are real numbers. Now

\[ \text{exp}(\sum a_\mu\gamma_\mu) = \text{exp}(\sum a_\mu\gamma_\mu) = \sum \text{exp}(a_\mu\gamma_\mu) = \sum b_\mu\gamma_\mu, \]

where $||b_\mu||$ is a 7x7 real orthogonal matrix with determinant = 1. It is easy to prove that, conversely, for every such $||b_\mu||$, there exist two sets of real $a_\mu$ satisfying (38). The group has thus a 2-1 homomorphism with $O_7$. [T. A. Tarski has pointed out to us that $O_7$ is called spin (7) in the standard language.]

In terms of the $\gamma$s, the element $R$ of $G_6$ is

\[ R = p_3 = -i\gamma_1\gamma_2\gamma_3 = \gamma_1\gamma_2\gamma_3. \]

Thus $R$ is an element of $O_7$, hence $G_6$ is a subgroup of $O_7$.

Both of the above two groups contain $G_6$ as a subgroup. There exist also groups that satisfy conditions (i) and (ii) but do not contain $G_6$ as a subgroup.

The group $SU_4$ was pointed out to us by Speiser and Tarski that the group $SU_4$ has an irreducible 8x8 representation which satisfies both conditions (i) and (ii). However, in this case it seems impossible to incorporate $\pi$ mesons and $K$ mesons without introducing more new bosons. It is clear that $G_6$ is not a subgroup of $SU_4$. The full implications and consequences of such possibilities still need to be investigated.

### APPENDIX B

We give in this Appendix explicit matrices for $G_1$, $R$, $\Xi$, $\Phi_1$, $\Phi_2$, and $N$ between the 16 states that describe a

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14 D. R. Speiser and J. A. Tarski (private communication).
single baryon or antibaryon:

$$\begin{bmatrix} B \\ B' \end{bmatrix}$$

where

$$\begin{bmatrix} \bar{p} \\ n \\ \Xi^- \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} \bar{p} \\ \bar{n} \\ \Xi^+ \end{bmatrix}$$

The antibaryon states\(^{18}\) are defined such that under the charge conjugation operation all baryon states \(\bar{p}, n, \Xi^-, \bar{\Xi}^+, \bar{Y}^+, Y^0, Z^0, Z^-\) are transformed in an identical way into their respective antibaryon states \(\bar{p}, \bar{n}, \bar{\Xi}^+, \Xi^-, \bar{Y}^+, Y^0, \bar{Z}^0, Z^-\). The minus signs in \(B'\) are so chosen that the matrices \(G, R, \Xi, \mathcal{M}, \mathcal{N}, \) and \(N\) are given by

$$G_1 = \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{bmatrix}$$

$$R_1 = \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{bmatrix}$$

$$N = \begin{bmatrix} -I & 0 & 0 & 0 \\ 0 & -I & 0 & 0 \\ 0 & 0 & -I & 0 \\ 0 & 0 & 0 & -I \end{bmatrix}$$

$$\mathcal{M}_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -I \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathcal{M}_4 = \frac{1}{2} I$$

$$\mathcal{N}_1 = \frac{1}{2} I$$

$$\mathcal{N}_2 = \frac{1}{2} I$$

$$\mathcal{N}_3 = \frac{1}{2} I$$

and

$$\mathcal{N}_4 = \frac{1}{2} I$$

where \(\sigma_i(i = 1, 2, 3)\) are the 2x2 Pauli spin matrices and \(I\) is the 2x2 unit matrix. All empty places in the above matrices are zeroes.

\(^{18}\) We use the notation that, e.g., \(\bar{Y}^+\) is the antiparticle of \(Y^+\) and is negatively charged.
CP INVARIANCE AND THE 2\pi DECAY MODE OF THE K^0

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In a recent experiment Christenson et al. 1) have found definite indications that the long lived component of the K^0, the K^0, decays into two \pi mesons

K^0 \rightarrow 2\pi.

(1)

The most direct interpretation of their experiment is to assume that CP invariance is violated 2) in this decay and hence in the weak interactions in general. However, it is important to realize that this experiment involves an entirely new domain of phenomena, those involving extremely small energies. The experiment of Christenson et al. is capable of detecting energy differences of the order of 10^{-8} eV. No other experiment in particle physics, up to now, has exhibited such sensitivity to small energies. On the other hand, none of the other experiments 3) on weak interactions that might have shown CP violation has done so. Therefore it is not unnatural to try to explain the new experiments in some other way. In this note we shall attempt to reconcile the two assumptions:

i) CP invariance holds for all interactions;

ii) the long lived component of the K^0, in (local)

vacuum, or at least in what is usually called "vacuum", decays into 2\pi's.

Up to now vacuum is particle physics has been taken to mean the absence of neighbouring material objects, although not necessarily that of long range fields such as gravitational or electromagnetic fields. These conventional fields, which are produced by distant bodies such as Milky Way and the other galaxies, are sufficiently well understood so that one can assume, with some confidence, that their effects on the K^0, \bar{K}^0 system can be neglected, either because they are too small (we have in mind the possible electromagnetic conversion between K^0 and \bar{K}^0 due to the difference in the charge radius of K^0 and \bar{K}^0) or because they have the same effect on particles and antiparticles (e.g., gravitational forces).

In this note we postulate that there is a new long range, extremely weak, field which can be neglected so long as one does not measure energies as small as those measured in the experiment of Christenson et al. This field, we assume, produces a potential energy which is equal in magnitude and opposite in sign for K^0 and \bar{K}^0, say \frac{1}{2}V for K^0 but -\frac{1}{2}V for \bar{K}^0. This new interaction is CP conserving and if we could maintain a given K^0 and \bar{K}^0 system, but transform the environment into its CP conjugate state, then this potential energy would reverse sign.

* During the completion of this paper, we learnt of a preprint by J. S. Bell and J. K. Perring, who have in-
As we shall see, this possibility leads to unique predictions concerning other "apparent" CP violating results which are in general quite different from the alternative possibility of assuming CP non-invariance of weak interactions. Our analysis does not depend on the precise nature of the new interaction except that it must produce a potential energy difference \( V \) between \( K^0 \) and \( \bar{K}^0 \). (To give an example, we may envisage that this potential energy difference is transmitted by a four-vector field similar to the electromagnetic interaction. It can be generated by any strongly interacting particle and has an amplitude proportional to, say, \( I_2 \), or \( \frac{3}{2}(N-S) \), of the particle where \( I_2 \) is the component of isotopic spin and \( N \) is baryon number, \( S \) is strangeness. (We leave open the question of whether it can have leptonic couplings).)

Whatever the origin of this interaction with macroscopic bodies, the fact that it gives rise to a static potential energy implies that so far as energies of the order of \( 10^{-8} \) eV can be neglected the decays of all particles other than neutral K mesons, there are no observable "apparent" CP non-invariant effects. For the neutral K meson, all other "apparent" CP non-invariant effects are completely determined by the parameter \( \nu \).

We first discuss the problem of a K meson in local isolation (i.e., in the absence of any neighbouring matter but taking into account the effects of the new long range interaction with distant objects). Let \( \psi(t) \) be the wave function of a K meson in the laboratory system. In what follows, we regard \( t \) as the independent variable and the co-ordinate \( r \) of the K meson as a dependent variable given by \( r = r_0 + vt \) where \( v \) is the K meson velocity and \( r_0 \) is its initial co-ordinate. The relative velocity of the laboratory and the distant macroscopic bodies will be neglected compared to the velocity of the K in the laboratory. We may write:

\[
\psi(t) = a(t) | K^0 \rangle + b(t) | \bar{K}^0 \rangle ,
\]

or simply

\[
\psi(t) = \langle \phi(t) \rangle
\]

where

\[
| K^0 \rangle = CP | \bar{K}^0 \rangle .
\]

The time development of \( \psi(t) \) can be determined from

\[
\frac{d\psi}{dt} = (\eta + \xi \sigma_x + \zeta \sigma_z)\psi
\]

where

\[
\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},
\]

and, for a locally isolated system, \( \eta, \xi, \zeta, \) are given by \( \hbar = c = 1 \)

\[
\eta = \eta_0 = \frac{1}{2\eta} [(m_1 + m_2) - \frac{1}{2} (\Gamma_1 + \Gamma_2)]
\]

\[
\xi = \xi_0 = \frac{1}{2\xi} [(m_1 - m_2) - \frac{1}{2} (\Gamma_1 - \Gamma_2)]
\]

and

\[
\zeta = \zeta_0 = \frac{1}{2} \nu .
\]

The subscripts 0 indicates that these are values for the K meson in a locally isolated system, and,

\[
\gamma = (1 - \nu^2)^{-\frac{1}{2}}
\]

where \( \nu \) is the velocity of the K meson in the laboratory. In the above equations, \( \eta \) and \( \zeta \) is the effect of the new interaction, \( m_1, \Gamma_1 \) and \( m_2, \Gamma_2 \) are the mass and width of \( K^0_1 \) and \( K^0_2 \) if \( \nu = 0 \). The solution of eq. (5) is well known. There are two eigensolutions \( \psi_1(t) \) and \( \psi_2(t) \) which are given by

\[
\psi_1 = (1 + |\epsilon|^2)^{-\frac{1}{2}} \left[ \psi_+ + \epsilon \psi_- \right] e^{i\lambda_1 t}
\]

and

\[
\psi_2 = (1 + |\epsilon|^2)^{-\frac{1}{2}} \left[ \psi_- - \epsilon \psi_+ \right] e^{i\lambda_2 t}
\]

where

\[
\psi_+ = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \quad \psi_- = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \epsilon = \xi^{-1} [(2 + \xi^2)^{\frac{1}{2}}]
\]

\[
\lambda_1 = \eta + (\xi^2 + \zeta^2)^{\frac{1}{2}}
\]

\[
\lambda_2 = \eta - (\xi^2 + \zeta^2)^{\frac{1}{2}}
\]

Using eqs. (7)-(9) and assuming small \( \nu \), we find

\[
\epsilon = \epsilon_0 \approx \frac{\gamma}{2} V [(m_1 - m_2) - \frac{1}{2} (\Gamma_1 - \Gamma_2)]^{-1} .
\]

The value \( \epsilon_0 \) for the experiment of Christenson et al. is found to be

\[
|\epsilon_0| \approx 2 \times 10^{-3}.
\]

It is important to note that if \( V \) is due to the average of the fourth component of a vector field, then \( V \) is independent of the energy of the K meson. Thus the parameter \( \epsilon_0 \) depends linearly on \( \gamma \). (On the other hand, \( V \) is proportional to \( \gamma^{-1} \) if it is due to a spin 0 field, and is proportional to \( \gamma \) if it is due to a spin 2 field. However, in the frame-
work of a simple local field theory, both spin 0 and spin 2 fields give equal static potential for a particle and its antiparticle. Therefore it seems more attractive to assume that this new interaction is due to a vector field with an extremely small mass.

The solution, eq. (11), also determines other "apparent" CP non-invariant effects in the K decay. For example, the ratio
\[
\frac{\text{rate } (K^0 \rightarrow \pi^+ + l^+ + \nu_l)}{\text{rate } (K^0 \rightarrow \pi^- + l^- + \bar{\nu}_l)}
\]
can be different from 1, where \(i = 1 \text{ or } 2\), and \(l = e \text{ or } \mu\). Assuming that the \(\Delta Q = \Delta S\) rule holds, we find
\[
\gamma_1 = \frac{1 + \epsilon_0}{1 - \epsilon_0} \frac{2}{\nu_l}
\]
and
\[
\gamma_2 = \frac{1 - \epsilon_0}{1 + \epsilon_0} \frac{2}{\nu_l}
\]
which gives \(|\gamma_1 - 1| \approx 6 \times 10^{-3}\) for the \(\epsilon_0\) given by eq. (16).

Next, we discuss the problem of a K meson in a medium. Let \(n\) and \(n'\) denote, respectively, the complex indices of refraction of \(K\) and \(\bar{K}\) in the medium. Eq. (5) remains valid where, instead of eqs. (7)-(9), the parameters \(\eta, \xi, \zeta\) are now given by
\[
\eta = \frac{1}{2\gamma} \left[ (n_1 + m_2) - i \frac{1}{2}(\Gamma_1 + \Gamma_2) \right] - \frac{1}{2}(n + n' - 2) k \nu_l \quad (18)
\]
\[
\xi = \frac{1}{2\gamma} \left[ (m_1 + m_2) - i \frac{1}{2}(\Gamma_1 - \Gamma_2) \right] \quad (19)
\]
\[
\zeta = \frac{3}{2\gamma} V - (n - n') k \nu_l \quad (20)
\]
and \(k\) is the momentum of the K meson.

The solution, eq. (11), also remains valid. For \(|\xi| \ll |\xi|\), the parameter \(\epsilon\) is given approximately by
\[
\epsilon \approx \frac{1}{2\gamma} V - (n - n') k \nu_l \left[ (m_1 + m_2) - i \frac{1}{2}(\Gamma_1 - \Gamma_2) \right]^{-1} \quad (21)
\]
The absolute magnitude of \(\epsilon\) in a medium can be obtained by measuring the corresponding rate of reaction (1). The interference term between \(V\) and \((n - n') k \nu_l\) in the expression for \(|\epsilon|\) can also determine the sign of \(V\).

It is interesting to speculate on the possible nature of such a long range interaction. As mentioned in the preceding discussion, a simple possibility is to make the ad hoc assumption of the existence of a four-vector field which interacts with all particles. For definiteness, let us assume that the effective "charge" for all strongly interacting particles is
\[
\mathbf{f} = \mathbf{I}_2
\]
or \(f \times \frac{1}{2}(N + S)\), where \(f\) is a dimensionless coupling constant. The potential energy difference \(V\) between \(K\) and \(\bar{K}\) due to the Milky Way is simply given by
\[
V = \frac{1}{2} f^2 (M_G/M_p) R^{-1} \quad (22)
\]
where \(M_G = \text{mass of the galaxy, } m_p = \text{mass of proton, } R = \text{effective radius of the galaxy,}\) defined so that \(G M_p R^{-1} = \text{gravitational potential on earth due to the galaxy, } G\) is the gravitation constant. The factor \(\frac{1}{2}\) is due to the isospin value of \(p\). It seems reasonable to assume that effects from very distant galaxies are small, and the main contribution is due only to the Milky Way. By taking \(M_G \approx 10^{11} \text{ solar mass, } R \approx 10 \text{ kpc and } V \approx 8 \times 10^{-4} \text{ cm}^{-1},\) we find
\[
f^2 \approx 2.5 \times 10^{-49} \quad (24)
\]
Such a long range force would also give rise to a slight difference between the observed "gravitational" mass and inertial mass *. The current experimental upper limit 6) on \(f^2\) is
\[
f^2 < 2 \times 10^{-45} \quad (25)
\]
which is certainly compatible with eq. (24).

Two of us (J. B. and T. D. L.) wish to thank Dr. S. F. Tuan for discussions, and Professors V. F. Weisskopf and L. Van Hove for the hospitality extended to us at CERN.

References
2) N. Cabibbo, Physica Letters 12 (1964) 137.
3) R. G. Sachs, to be published.

* We implicitly assume that the mass of this vector field is smaller than, or comparable to, \(R^{-1}\).

** The reasoning here is very similar to that given by Lee and Yang*.2)
Symmetry Properties of Leptons in the Zero-Mass Limit (*)

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Summary. — The symmetry properties of all known leptons are studied for those processes in which their mass differences can be neglected. It is found that in this approximation both the electromagnetic interaction $H_{\text{elect}}$ and the weak interaction $H_{\text{weak}}$ are invariant under a $U_3 \times U_2$ group of transformations. Thus, in the zero-mass limit, all lepton systems are characterized by six quantum numbers (in addition to charge, momentum, etc.). The electromagnetic interaction is known to be further invariant under a discrete symmetry operator $C$ (charge conjugation), or $P$ (parity). It is proposed that the weak interaction is also invariant under a discrete symmetry operator, called $D$, which exchanges charged leptons and neutral leptons. The $C$ symmetry is violated by $H_{\text{elect}}$, and the $D$ symmetry is violated by $H_{\text{weak}}$ and by the mass differences. Applications of these symmetry properties to high-energy lepton reactions are discussed.

1. — Introduction.

It is well known that there exists a remarkable symmetry between muons and electrons if their mass difference $(m_\mu - m_e)$ is neglected. It has been shown by Feinberg and Gürsey (*) that in this approximation the free Hamiltonian, the electromagnetic interaction and the weak interaction are all invariant under a $U_3$ group of transformations among the leptons. The purpose of this note is to make a more complete study of the symmetry properties of leptons by neglecting the mass differences between all leptons. As will be shown in

(*) Work supported in part by the U.S. Atomic Energy Commission.  
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the next Section, in the zero-mass limit all known interactions are invariant under a $U_q \times U_s$ group. Thus, all lepton systems are characterized by six quantum numbers (in addition to charge, momentum, etc.). The application of this symmetry property leads to a number of relations between different leptonic processes which are valid to all orders in the electromagnetic and the weak interactions, provided the zero-mass limit is a good approximation.

The question of zero-mass limit in electrodynamics has been investigated in the literature (2,3). If radiative corrections are included, the power series expansions of most differential cross-sections contain logarithmic singularities as the mass of the charged particle $\rightarrow 0$. The underlying reason for having such a mass singularity is the appearance of degenerate states in the zero-mass limit. Similarly to the infra-red divergence, these singularities can be completely removed for every term in the power series expansion of any transition probability, provided it is averaged over an appropriate ensemble of degenerate states. For these averages, the zero-mass limit becomes a good approximation if

$$\frac{m_t}{E} \ll 1$$

is satisfied, where $m_t$ is the mass of the charged lepton and $E$ is, for most processes, simply the energy of the lepton. As discussed in refs. (2) and (4), for any given Feynman graph the evaluation of its mass singularity is a relatively simple task. Thus, the zero-mass symmetry properties can also be applied to each individual graph which contains mass singularity, provided that the singular part is calculated separately and eq. (1) is satisfied.

At present, there exists no fundamental theory of weak interactions. From all existing weak reactions, it appears that, so far as the leptons are concerned, only the combination

$$j_\lambda = i \bar{\psi}_e \gamma_\lambda (1 + \gamma_5) \psi_e + i \bar{\psi}_\mu \gamma_\lambda (1 + \gamma_5) \psi_\mu$$

occurs in the weak interaction Hamiltonian $H_{\text{weak}}$, where $\psi_e$, $\psi_\mu$, $\psi_\tau$, $\psi_\nu$ denote, respectively, the field operators for the leptons $e^-$, $\mu^-$, $\tau^-$, $\nu_e$, $\nu_\mu$, and $\gamma_1, \ldots, \gamma_5$ are the usual five anticommuting Hermitian matrices. In the following, we consider the general case

$$H_{\text{weak}} = \text{arbitrary function of } j_\lambda \text{ and } j_\lambda'$$

and make no further restriction about its specific form.

Lacking a complete theory, it is not possible to make definite statements concerning the properties of mass-singularity in weak interactions. However, from the general discussions given in ref. (1), and the form of $H_{\text{weak}}$ given by eq. (3), it is expected that to arbitrary high orders in weak interactions there also should exist no mass singularity in the appropriate ensemble averages, provided the ultra-violet divergence difficulties are solved. Furthermore, since the symmetry property is valid to all orders in all interactions, its applications to high-energy leptonic processes may serve as a test of our basic assumption eq. (3).

2. – Symmetry properties.

The Hamiltonian density $H$ of the known leptons can be written as

$$H = H_{\text{free}} + H_{\text{elec}} + H_{\text{weak}},$$

In order to study the zero-mass limit, it is useful to decompose the free particle Hamiltonian density $H_{\text{free}}$ into two terms

$$H_{\text{free}} = H_e + H_\mu,$$

where

$$H_\mu = m_e \nu_e^+ \gamma_4 \nu_e^+ + m_\mu \nu_\mu^+ \gamma_4 \nu_\mu^+.$$

The weak interaction part of the Hamiltonian density is given by eq. (3), and the electromagnetic part is given by

$$H_{\text{elec}} = e J^a_A A^a_A,$$

where

$$J^a_A = i \psi^+_e \gamma_4 \gamma_A \psi_e^+ + i \psi^+_\mu \gamma_4 \gamma_A \psi_\mu^+,$$

and $A^a_A$ describes the electromagnetic field.

In eq. (2), the neutrino fields can be chosen to satisfy

$$\gamma_4 \nu_l = \nu_l,$$

where $l = e$ or $\mu$. Similarly, the charged lepton fields $\nu_i$ can be decomposed into

$$\nu_i = \nu_{i,L} + \nu_{i,R}.$$
where

\[ \psi_{iL} = \frac{1}{2}(1 + \gamma_3) \psi_i \]

\[ \psi_{iR} = \frac{1}{2}(1 - \gamma_3) \psi_i. \]

**Theorem.** - The Hamiltonian density \((H_0 + H_{\text{elec}} + H_{\text{weak}})\) is invariant under a \(U_2 \times U_2\) group which transforms

\[
\begin{align*}
\varphi_L &\equiv \begin{pmatrix} \psi_{eL} \\ \psi_{eL} \end{pmatrix} \rightarrow u \varphi_L, \\
\varphi_R &\equiv \begin{pmatrix} \psi_{eR} \\ \psi_{eR} \end{pmatrix} \rightarrow v \varphi_R,
\end{align*}
\]

but leaves the field operators of all other particles invariant. The matrices \(u\) and \(v\) are two arbitrary \((2 \times 2)\) unitary matrices.

This theorem can be easily proved by noticing that both \(J_\lambda\) and \(j_\lambda\) are invariant under these two independent \(U_2\) transformations. The special case \(u = v\) has been discussed by FEINBERG and GÜRSEY (1).

From this theorem, it follows that \((H_0 + H_{\text{elec}} + H_{\text{weak}})\) commutes with the following eight Hermitian operators:

\[
\begin{align*}
N_L &= \int (\bar{\varphi}_L \varphi_L + \bar{\varphi}_R \varphi_R) \, d^2r, \\
N_R &= \int (\bar{\varphi}_L \varphi_R + \bar{\varphi}_R \varphi_L) \, d^2r, \\
L_i &= \frac{1}{2} \int (\bar{\varphi}_L \sigma_i \varphi_L + \bar{\varphi}_R \sigma_i \varphi_R) \, d^2r,
\end{align*}
\]

and

\[
R_i = \frac{1}{2} \int (\bar{\varphi}_R \sigma_i \varphi_R) \, d^2r,
\]

where the \(\sigma_i (i = 1, 2, 3)\) are the usual three anticommuting \((2 \times 2)\) Pauli matrices. These operators obey the following commutation relations:

\[
\begin{align*}
[L_a, L_b] &= i \epsilon_{abc} L_c, \\
[R_a, R_b] &= i \epsilon_{abc} R_c,
\end{align*}
\]
where $\varepsilon_{abc} = +1$ or $-1$, depending on whether $(a, b, c)$ is an even or odd permutation of $(1, 2, 3)$; otherwise, $\varepsilon_{abc} = 0$. All other commutators between these eight operators are zero. In the zero-mass limit, all lepton systems are characterized by six quantum numbers (in addition to charge, momentum, etc.):

\begin{equation}
N_L, \ N_R, \ L, \ L_a, \ R \text{ and } R_a,
\end{equation}

where $N_L, \ N_R, \ L_a$ and $R_a$ denote the eigenvalues of the corresponding operators, $L(L + 1)$ and $R(R + 1)$ are, respectively, the eigenvalue of the operators $L^z$ and $R^z$. All nonleptons are also eigenstates of these operators but with eigenvalues $= 0$.

The mass term $H_m$ breaks the symmetry. The full Hamiltonian $H$ commutes only with

\begin{equation}
N \equiv N_L + N_R
\end{equation}

and

\begin{equation}
M \equiv L_2 + R_2,
\end{equation}

which are the two familiar leptonic numbers that remain conserved in the presence of the mass term. From the discussion given in the introduction, we expect the zero-mass limit to be important for any high-energy leptonic processes. The use of these operators $L, R,$ etc., gives a number of relations between the amplitudes for different leptonic processes.

3. – Applications.

It is convenient to classify the twelve single-lepton states into two $L = \frac{1}{2}$ doublets with $N_L = 1$ ($N_R = R = 0$)

\begin{equation}
\begin{pmatrix}
\varepsilon_L \\
\mu_L
\end{pmatrix}
\text{ and } \begin{pmatrix}
\nu_e \\
\nu_\mu
\end{pmatrix},
\end{equation}

two $L = \frac{1}{2}$ doublets with $N_L = -1$ ($N_R = R = 0$)

\begin{equation}
\begin{pmatrix}
\bar{\mu}_L \\
-\bar{\nu}_L
\end{pmatrix}
\text{ and } \begin{pmatrix}
\bar{\nu}_\mu \\
-\bar{\tau}_e
\end{pmatrix},
\end{equation}

one $R = \frac{1}{2}$ doublet with $N_R = 1$ ($N_L = L = 0$)

\begin{equation}
\begin{pmatrix}
\varepsilon_R \\
\mu_R
\end{pmatrix}
\end{equation}
and one \( R = \frac{1}{2} \) doublet with \( N_R = -1 \) \( (N_L = L = 0) \)

\[
\begin{pmatrix}
\bar{\mu}_R \\
-\bar{e}_R
\end{pmatrix}
\]

where \( l_s \) (\( l = e \) or \( \mu \)) denotes the charged lepton \( l^- \) with helicity \( s \) and \( \bar{l}_s \) denotes its antiparticle (i.e., one with opposite charge and opposite helicity). A particularly useful operator is

\[
S = \exp[\pm \pi (L_z + R_z)],
\]

which exchanges the upper and the lower members of the doublets.

For simple reactions, the use of this exchange operator and the four conservation laws of \( N_L, N_R, L_z, R_z \) are sufficient to obtain almost all physically interesting results. For example, let us consider either the reaction

\[
l_1(\mathbf{q}) + A \rightarrow l_2(\mathbf{p}) + B,
\]

or

\[
A + B \rightarrow l_1(\mathbf{q}) + l_2(\mathbf{p}) + C,
\]

where \( A, B, C \) denote arbitrary systems of nonleptons and \( l_1(\mathbf{q}) \) [or \( l_2(\mathbf{p}) \)] can be any one of the twelve single lepton states with momentum \( \mathbf{q} \) (or \( \mathbf{p} \)).

Let \( a[l_1; l_2] \) be the-zero mass limit of the amplitude for reaction (27). The leptons \( l_1 \) and \( l_2 \) must have the same quantum numbers \( N_L, N_R, L, L_z \) and \( R, R_z \); otherwise, \( a[l_1; l_2] = 0 \). The use of \( S \) relates the amplitude for \( l_1 = \uparrow \) with that for \( l_1 = \downarrow \) member of the same doublet \((\dagger)\). Thus, we find

\[
a[l_1(\uparrow); l_2(\uparrow)] = a[l_1(\downarrow); l_2(\downarrow)].
\]

For example, if \( l_1(\uparrow) = v_e \) and \( l_2(\uparrow) = e_L \) then eq. (29) becomes

\[
a[v_e; e_L] = a[v_\mu; \mu_L].
\]

Similar expressions can easily be written down for all other choices of \( l_1(\uparrow) \) and \( l_2(\uparrow) \).

\((\dagger)\) Throughout this Section, we use \( l_1(\uparrow) \) and \( l_1(\downarrow) \) to denote, respectively, the two \( L_3 = + \frac{1}{2} \) and \( L_3 = - \frac{1}{2} \) members (or, \( R_3 = + \frac{1}{2} \) and \( R_3 = - \frac{1}{2} \) members) of any \( L = \frac{1}{2} \) doublet (or, \( R = \frac{1}{2} \) doublet).
In an identical way, we can analyse reaction (28). Let \( b[l_1, l_2] \) be the zero-

mass limit of its amplitude. The two leptons must have the same \( L \) and \( R \),

but their \( N_L, N_R, L_0 \) and \( R_0 \) must be of opposite signs; otherwise, \( b[l_1, l_2] = 0 \).

The further use of \( S \) leads to the identity (*):

\[
(31) \quad b[l_1(\uparrow), l_2(\downarrow)] = -b[l_1(\downarrow), l_2(\uparrow)].
\]

For example, if \( l_1(\uparrow) = e_L \) and \( l_2(\downarrow) = -\bar{e}_L \), eq. (31) becomes

\[
(32) \quad b[e_L, \bar{e}_L] = b[\mu_L, \bar{\mu}_L].
\]

Next, we discuss the reaction

\[
(33) \quad l(q) + A \rightarrow l_1(p) + l_2(p') + l_3(p'') + B,
\]

where \( A \) and \( B \) are two arbitrary systems of nonleptons. Let the amplitude

of reaction (33) in the zero-mass limit be \( c[l_1; l_1, l_2, l_3] \). For any given initial

state \( l \), the four conservation laws of \( N_L, N_R, L_0, R_0 \) limit the number of possible

choices of \( l_1, l_2, l_3 \) for which the amplitude is not zero. These choices can be

further classified into two classes:

\( \alpha \) the leptons \( l, l_1, l_2, l_3 \) all have the same quantum number \( L = \frac{1}{2} \), or

all have \( R = \frac{1}{2} \). By using the properties of \( L \) and \( R \), we find

\[
(34) \quad c_\alpha[l(\uparrow); l_1(\uparrow), l_2(\uparrow), l_3(\downarrow)] + c_\alpha[l(\uparrow); l_1(\downarrow), l_2(\downarrow), l_3(\uparrow)] +
\]

\[+ c_\alpha[l(\uparrow); l_1(\downarrow), l_2(\uparrow), l_3(\uparrow)] = 0,
\]

where the subscript \( \alpha \) indicates that eq. (34) is valid only for case \( \alpha \). The use

of \( S \) leads to

\[
c[l(\uparrow); l_1(\uparrow), l_2(\downarrow), l_3(\downarrow)] = -c[l(\downarrow); l_1(\downarrow), l_2(\downarrow), l_3(\uparrow)],
\]

\[
c[l(\uparrow); l_1(\uparrow), l_2(\downarrow), l_3(\downarrow)] = -c[l(\downarrow); l_1(\downarrow), l_2(\downarrow), l_3(\downarrow)],
\]

and

\[
(35) \quad c[l(\uparrow); l_1(\downarrow), l_2(\uparrow), l_3(\downarrow)] = -c[l(\downarrow); l_1(\downarrow), l_2(\uparrow), l_3(\downarrow)],
\]

\( \beta \) Two of the leptons, say \( l \) and \( l_1 \), have the same quantum number

\( L = \frac{1}{2} \) and other two \( l_2 \) and \( l_3 \) have \( R = \frac{1}{2} \). (Or, \( l, l_1 \) have \( R = \frac{1}{2} \) and \( l_2, l_3 \) have

\( L = \frac{1}{2} \)). In this case we have, instead of eq. (34).

\[
(36) \quad c_\beta[l(\uparrow); l_1(\uparrow), l_2(\downarrow), l_3(\downarrow)] = -c_\beta[l(\uparrow); l_1(\uparrow), l_2(\downarrow), l_3(\downarrow)],
\]

\( \Xi \)
where the subscript $\beta$ indicates that it holds only for case $\beta$). The equations listed in (35) remain valid.

As an example, the complete set of identities for $l = e_L$ is given below (arranged in descending order of practical importance):

\begin{align}
(37) & \quad \sigma[e_L; e_L, \bar{e}_L, \bar{e}_L] = \sigma[e_L; e_L, \mu_L, \bar{\mu}_L] + \sigma[e_L; \mu_L, e_L, \bar{\mu}_L], \\
(38) & \quad \sigma[e_L; e_L, e_R, \bar{e}_R] = \sigma[e_L; e_L, \mu_R, \bar{\mu}_R], \\
(39) & \quad \sigma[e_L; e_L, \bar{e}_L, \bar{\nu}_L] = \sigma[e_L; e_L, \mu_L, \bar{\nu}_L] + \sigma[e_L; \mu_L, e_L, \bar{\nu}_L], \\
(40) & \quad \sigma[e_L; e_L, e_R, \bar{\nu}_L] = \sigma[e_L; e_L, \mu_R, \bar{\nu}_L] + \sigma[e_L; \mu_R, e_L, \bar{\nu}_L], \\
(41) & \quad \sigma[e_L; e_L, \bar{e}_R, \bar{\nu}_R] = \sigma[e_L; e_L, \mu_R, \bar{\nu}_R], \\
(42) & \quad \sigma[e_L; e_L, e_R, \bar{\nu}_R] = \sigma[e_L; e_L, \mu_R, \bar{\nu}_R] + \sigma[e_L; \mu_L, e_L, \bar{\nu}_R], \\
(43) & \quad \sigma[e_L; e_R, e_R, \bar{\nu}_L] = \sigma[e_L; e_R, \mu_L, \bar{\nu}_L] + \sigma[e_L; \mu_L, e_R, \bar{\nu}_L], \\
(44) & \quad \sigma[e_L; e_R, e_R, \bar{\nu}_R] = \sigma[e_L; e_R, \mu_R, \bar{\nu}_R] + \sigma[e_L; \mu_R, e_R, \bar{\nu}_R],
\end{align}

and all other unrelated amplitudes $\sigma[e_L; l_1, l_2, l_4] = 0$.

These identities are valid for arbitrary nonleptonic systems $A$ and $B$. For different identities, $A$ and $B$ can (and sometimes must) be different. All above identities are also valid for any given momentum distribution. The different momentum distributions are related by the antisymmetric relation

\begin{align}
(45) & \quad \sigma[l; l_1(p), l_2(p'), l_4(p'')] = -\sigma[l; l_2(p'), l_4(p), l_3(p'')] = \\
& \quad \quad = -\sigma[l; l_1(p), l_2(p''), l_3(p')].
\end{align}

In eqs. (34)–(44), all amplitudes $\sigma[l; l_1, l_2, l_4]$ refer to that for the momentum distribution given by (33). Thus e.g., $\sigma[e_L; e_L, \mu_L, \bar{\mu}_L]$ denotes $\sigma[e_L; e_L(p), \mu_L(p'), \bar{\mu}_L(p'')]$, which for $p \neq p'$ is not the same as

\[-\sigma[e_L; \mu_L, e_L, \bar{\mu}_L] = -\sigma[e_L; \mu_L(p), e_L(p'), \bar{\mu}_L(p'')].

As a final example, we discuss the reaction

\begin{align}
(46) & \quad l_1(q) + l_2(-q) \rightarrow l_3(p) + l_4(p') + A,
\end{align}

where $A$ is, again, any system of nonleptons. Let $d[l_1, l_2, l_3, l_4]$ be the zero-mass limit of the amplitude for reaction (46). As before, the conservation laws of $N_L$, $N_R$, $L_3$ and $R_3$ limit the different choices of lepton states. These states can be further classified as follows:
\(\alpha\) The four leptons \(l_1, l_2, l_3, l_4\) all have the same quantum number \(L = \frac{1}{2}\) or all have \(R = \frac{1}{4}\). By using the symmetry properties, we have

\[
\begin{align*}
d_a[l_1(\uparrow), l_2(\downarrow); l_3(\uparrow), l_4(\downarrow)] + d_a[l_1(\uparrow), l_2(\downarrow); l_3(\downarrow), l_4(\uparrow)] - \\
d_a[l_1(\downarrow), l_2(\uparrow); l_3(\uparrow), l_4(\downarrow)] = 0.
\end{align*}
\]

For example, when \(l_1(\uparrow) = l_2(\uparrow) = e_\uparrow\) and \(l_3(\uparrow) = l_4(\uparrow) = \bar{\mu}_\uparrow\), eq. (47) becomes

\[
d[e_L, \bar{e}_L; e_L, \bar{e}_L] - d[e_L, \bar{e}_L; \mu_L, \bar{\mu}_L] - d[e_L, \bar{\mu}_L; e_L, \bar{e}_L] = 0.
\]

\(\beta\) The two initial leptons \(l_1, l_2\) have the same quantum number \(L = \frac{1}{2}\), but the two final ones \(l_3, l_4\) both have \(R = \frac{1}{2}\); or, the initial leptons both have \(R = \frac{1}{4}\) and the final ones both have \(L = \frac{1}{4}\). In this case, we have

\[
d_p[l_1(\uparrow), l_2(\downarrow); l_3(\uparrow), l_4(\downarrow)] = -d_p[l_1(\uparrow), l_2(\downarrow); l_3(\downarrow), l_4(\uparrow)].
\]

\(\gamma\) One of the initial leptons, say \(l_1\), has \(L = \frac{1}{2}\) and the other one \(l_2\) has \(R = \frac{1}{4}\). Thus, one of the final leptons, say \(l_3\), must have \(L = \frac{1}{4}\) and the other one \(l_4\) has \(R = \frac{1}{2}\). We find

\[
d_p[l_1(\uparrow), l_2(\downarrow); l_3(\uparrow), l_4(\downarrow)] = -d_p[l_1(\uparrow), l_2(\downarrow); l_3(\downarrow), l_4(\uparrow)].
\]

Here the subscripts \(\alpha, \beta, \gamma\) indicate the cases in which these equations are valid.

The application of \(S\) leads to the following identities which are valid for all cases:

\[
\begin{align*}
d[l_1(\uparrow), l_2(\downarrow); l_3(\downarrow), l_4(\downarrow)] &= d[l_1(\downarrow), l_2(\downarrow); l_3(\downarrow), l_4(\downarrow)], \\
d[l_1(\uparrow), l_2(\downarrow); l_3(\downarrow), l_4(\downarrow)] &= d[l_1(\downarrow), l_2(\downarrow); l_3(\downarrow), l_4(\downarrow)],
\end{align*}
\]

and

\[
\begin{align*}
d[l_1(\uparrow), l_2(\downarrow); l_3(\downarrow), l_4(\downarrow)] &= d[l_1(\downarrow), l_2(\downarrow); l_3(\downarrow), l_4(\downarrow)].
\end{align*}
\]

All these relations are valid to all orders of \(H_{\text{int}}\) and \(H_{\text{weak}}\). They can be applied to any Feynman graph (or sum of graphs) for a high-energy leptonic process if it does not contain mass singularity or if its mass singularity has already been separated out.

4. – Discrete symmetries.

Throughout this paper, we study only those symmetry properties of leptons that are independent of space and time. In the zero-mass limit, there are twelve
field operators: \( \psi_{\nu}, \psi_{\mu}, \psi_\epsilon, \psi_\mu, \psi_\mu, \psi_{\mu,R} \) and their conjugate operators \( \psi^*_\nu, \psi^*_\mu, \psi^*_\epsilon, \psi^*_\mu, \psi^*_\mu, \psi^*_{\mu,R} \). In the representation \( \gamma_1, \gamma_2, \gamma_3 \) are real and \( \gamma_4, \gamma_5 \) are imaginary, the conjugate field operator of \( \psi_i \) is related to its Hermitian conjugate \( \psi^+_i \) by

\[
(\psi^+_i)_a = (\psi^*_i)_a ,
\]

where \( a = 1, 2, 3, 4 \), and \( t = (\nu), \) or \( (\nu_i), \) or \( (\epsilon, L), \) ..., or \( (\mu, R) \). The six operators \( \psi_\nu, \psi_\mu, \psi_\epsilon, \psi_\mu, \psi_\mu, \psi_{\mu,R} \) satisfy the same set of anticommutation relations (which is different from that satisfied by \( \psi^*_\nu, \psi^*_\mu, \psi^*_\epsilon, \psi^*_\mu, \psi^*_\mu, \psi^*_{\mu,R} \)). The quantization rules and \( H_{\text{e}} \) are invariant under a \( U_6 \) group of transformations among these six field operators at the same space time point. This \( U_6 \) symmetry is broken by the interactions of leptons.

It can be shown that the only subgroup of \( U_6 \) which leaves the currents \( J_\lambda \) and \( j_\lambda \) invariant is the \( U_2 \times U_2 \) group given by eqs. (11) and (12), according to which these six operators are grouped into three doublets, \( \varphi_L, \varphi, \) and \( \varphi^*_R \). We know that the electromagnetic interaction is invariant under charge conjugation \( C \) which connects \( \varphi_L \) with \( \varphi^*_R \),

\[
C \varphi_L C^+ = \varphi^*_R
\]

and

\[
C^+ = 1 .
\]

The electromagnetic current changes its sign under \( C \), but

\[
C H_{\text{e}} C^+ = H_{\text{e}} .
\]

The charge conjugation symmetry is broken by the weak interaction. The similarity in the roles of the three doublets \( \varphi_L, \varphi, \) and \( \varphi^*_R \) leads one to introduce another discrete operator \( D \), defined by

\[
D \varphi_L D^+ = \varphi
\]

and

\[
D^+ = 1 .
\]

The leptonic current \( j_\lambda \) transforms into \( j^*_\lambda \) under \( D \); i.e.,

\[
D j_\lambda D^+ = j^*_\lambda ,
\]

where \( j^*_\lambda = j^*_\lambda \) for \( \lambda \neq 4 \), but \(- j^*_\lambda = j^*_\lambda \) for \( \lambda = 4 \). It is natural to impose the condition

\[
D H_{\text{weak}} D^+ = H_{\text{weak}} .
\]

The \( D \) symmetry is broken by the electromagnetic interaction.
The symmetry property of C requires that all (leptonic or nonleptonic) electromagnetic currents should change sign under C. Similarly, D symmetry (eq. (57)) requires that all nonleptonic currents in weak interactions (*) should obey a similar transformation under D as that given by eq. (56).

The D symmetry leads to identities which, e.g., equate the amplitude for reaction

\[(58) \quad \nu_e + A \rightleftharpoons e_L + B\]

with that for

\[(59) \quad e_L + A^p \rightleftharpoons \nu_e + B^p,\]

in the zero-mass limit, where A and B are arbitrary systems of nonleptons, and \(A^p, B^p\) are their respective D-conjugate system. For example, if \(A = n, B = p + \pi^0\), then \(A^p = p\) and \(B^p = n + \pi^0\). The consequence of D symmetry (*) is valid to all orders of \(H_{\text{weak}}\), but is violated by \(H_{\text{elct}}\) and by the mass differences.

***

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(*) This implies a symmetry requirement also for the strongly interacting particles, which can be satisfied in many models of strong interactions. A full discussion will be given elsewhere.

(*) To lowest order in weak interaction and for leptonic interaction with nonstrange particles, D symmetry is the same as the \(|\Delta I| = 1\) rule. See T. D. LEE and C. N. YANG: Phys. Rev., 119, 1410 (1960).

RIASSUNTO (*)

Si studiano le proprietà di simmetria di tutti i leptoni noti per quei processi in cui si possono trascuare le loro differenze di massa. Si trova che in questa approssimazione sia l'interazione elettromagnetica \(H_{\text{elct}}\) che l'interazione debole \(H_{\text{weak}}\) sono invarianti rispetto a un gruppo di trasformazioni \(U_3 \times U_2\). Così, nel limite di massa zero, tutti i sistemi leptonici sono caratterizzati da sei numeri quantici (oltre alla carica, l'impulso ecc.). È noto, inoltre, che l'interazione elettromagnetica è invariante rispetto a un operatore di simmetria discreto C (coningazione della carica) o P (parità). Si postula che l'interazione debole sia anch'essa invariante rispetto ad un operatore di simmetria discreto, D, che scambia i leptoni carichi con leptoni neutrali. La simmetria C è violata da \(H_{\text{elct}}\) e la simmetria D da \(H_{\text{weak}}\) e dalle differenze di massa. Si discutono le applicazioni di queste proprietà di simmetria alle reazioni dei leptoni di alta energia.

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Hypercharge Conservation, CP Invariance and the Possible Existence of a Zero-Mass Zero-Spin Field*

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It is shown that by introducing a neutral zero-mass zero-spin field $\phi$, the Lagrangian density of all interactions (including weak interactions) can be made invariant under the hypercharge gauge transformation. Consequently, there exists a hypercharge current density $J_\mu$ that is absolutely conserved. The current density $J_\mu$ is related to the usual hypercharge current density $j_\mu$ of all the presently known particles by

$$J_\mu = j_\mu - \lambda^{-1} \partial x_\mu \partial_s \phi,$$

where $\lambda$ is a coupling parameter. The same Lagrangian density is invariant under CP transformation, time-reversal transformation, and Lorentz transformation. It turns out that if the conserved quantity $\int j_\mu \partial x_\mu \partial_s \phi$ exists, then there exists an energy difference between any hypercharged particle and its antiparticle with the same momentum. Such an energy difference would induce decays such as $K^0 \rightarrow 2\pi$, and the decay rate is proportional to the square of the $K$-meson energy.

1. INTRODUCTION

It is well-known that the strong and electromagnetic interactions conserve the hypercharge $Y$ and are invariant under the hypercharge gauge transformation

$$\psi_a(t, \xi) \rightarrow e^{iY a \theta} \psi_a(t, \xi),$$

where $\theta$ is an arbitrary constant, $Y_a = 0$, or $\pm 1$, $\cdots$ is the hypercharge of the particle $a$, and $\psi_a$ is its field operator. The weak interaction allows processes such as

$$A^\pm \rightarrow p^+ + \pi^-,$$

where $Y_A = Y_\pi = 0$ and $Y_p = 1$, it follows that reaction (2) violates $Y$ conservation, and thus the weak interaction is usually regarded to be noninvariant under the hypercharge gauge transformation.

Recently, Christenson, Cronin, Fitch, and Turlay observed that the long-lived component of the neutral $K$-meson, the $K^\circ_0$, decays into two $\pi$ mesons

$$K^0 \rightarrow \pi^+ + \pi^-$$

which apparently indicates that $CP$ is also not conserved in the weak interaction. The $CP$ invariance is connected with the transformation

$$\psi_a(t, \xi) \rightarrow \eta_a \bar{\psi}_a(-t, \xi),$$

where $\eta_a$ is a phase factor and $\bar{\psi}_a$ is the field operator for the antiparticle of $a$. The experiment by Christenson et al. seems to imply that the weak interaction might not be invariant under the transformation (4).

The purpose of this paper is to point out that by introducing a neutral field, $\phi$ which is associated with a zero-rest mass and zero-spin particle, it is possible to preserve both the hypercharge gauge invariance and the $CP$ invariance for the weak interaction, and at the same time allow all the observed weak reactions such as (2) and (3).

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To see how this can be achieved we may consider, for example, reaction (2) and represent the relevant part of the usual weak-interaction Lagrangian density by (in the absence of $\phi$)

$$G[\beta A \pi] + \text{H.c.},$$

(5)

where $G$ is the appropriate weak-coupling constant and

$$G[\beta A \pi] = \psi^\dagger \gamma \gamma_3 \gamma_5 (1 + b)$

$$\partial x_\mu \partial_s \phi,$$

(6)

the $\gamma_i, \gamma_2, \cdots \gamma_4$ are the usual Dirac matrices, $b$ = constant, and the dagger denotes Hermitian conjugation. The expression (5) is not invariant under the hypercharge gauge transformation. In order to maintain the gauge invariance, we replace (5) by

$$G[\beta A \pi] \exp (-i \lambda \phi) + \text{H.c.},$$

(6)

and assume that under the hypercharge gauge transformation (1), the field $\phi$ transforms according to

$$\phi \rightarrow \phi - \lambda^{-1} \theta,$$

(7)

where the parameter $\lambda$ is real and has the dimension of a length. The gauge invariance requires that $\phi$ have zero mass. Under the $CP$ transformation, we assume that

$$\phi(t, \xi) \rightarrow -\phi(-t, \xi),$$

(8)

The new Lagrangian density (6) is, then, invariant under the Lorentz transformation, the hypercharge gauge transformation, and the $CP$ transformation.

As a consequence of the hypercharge gauge invariance, there exists a conserved hypercharge current density

$$J_\mu = j_\mu - \lambda^{-1} \partial x_\mu \partial_s \phi$$

(9)

which satisfies the conservation law

$$\frac{\partial j_\mu}{\partial x_\mu} = 0,$$

(10)

where $j_\mu$ is the usual hypercharge current density due to all the presently known particles. For example, in

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the case of fermions such as $\bar{p}, n, \Xi$, etc., we have
\begin{equation}
J_\mu^p = i \sum_a Y_{\alpha_a} \gamma^\nu \gamma^\mu \psi_a^\dagger \phi^\nu \psi_a^a.
\end{equation}

Equation (10) implies that
\begin{equation}
\int d^3x \frac{\partial \phi}{\partial t} = i \mathcal{Q}
\end{equation}
is absolutely conserved, and its value is determined by the initial condition of the system. Decays such as $\Lambda^o \rightarrow \bar{p} + \pi^-$ do not violate the conservation of hypercharge current $J_\mu^p$, provided we also include in the definition of $J_\mu^p$ the contribution of the $\phi$ field.

It will be shown that if the system is in any state with $Q \neq 0$, there must exist a difference of energy between any hypercharge particle and its antiparticle of the same momentum. For $K^0$ and $\bar{K}^0$, this energy difference is given by
\begin{equation}
V = -2\pi^2(Q/\Omega),
\end{equation}
where $\Omega$ is the volume of the system and is, presumably, the same as the volume of the “universe.” Under CP transformation, $J_\mu$ changes sign. The CP invariance implies that, for every eigenstate (of the entire system) with $Q = Q_0$, there exists another eigenstate of the same energy but with $Q = -Q_0$. The existence of this energy difference $V$ between $K^0$ and $\bar{K}^0$ in a given state of the system, therefore, does not contradict the requirement of CP invariance.

It has been pointed out that, independent of the precise nature of the mechanism, if such an energy difference $V$ exists between $K^0$ and $\bar{K}^0$ then the decay $K^0 \rightarrow 2\pi$ can occur, and its rate is proportional to
\begin{equation}
|\epsilon|^2,
\end{equation}
where $(\hbar = c = 1)$
\begin{equation}
\epsilon \equiv \frac{1}{2} \gamma^\nu \left[ (m_1 - m_2) - i \frac{1}{2} (\Gamma_1 - \Gamma_2) \right] - i \frac{1}{2} (\Gamma_1 - \Gamma_2); \end{equation}
and $s$ is the velocity of $K^0$ with respect to the rest system of the “universe.” Since in the present theory, the energy difference $V$ is independent of $\gamma$, the observed rate for the $2\pi$ decay of $K^0$ must be proportional to $\gamma^2$.

Identical considerations can, of course, be applied to other quantities such as strangeness, or $I_\mu$. The hypercharge is used only as a representative of any such quantities that could be conserved by introducing a zero-mass and zero-spin field. Throughout this paper we follow the method of canonical formalism of quantum-field theory. The effect of general relativity is not discussed.

\section{2. LAGRANGIAN AND HAMILTONIAN}

In general, we can write the usual weak-interaction Lagrangian density in the absence of the $\phi$ field as
\begin{equation}
L_{\text{weak}}(0) = G \sum \Delta Y,
\end{equation}
where the subscript $\Delta Y$ denotes the amount of hypercharge violation and the sum extends over $\Delta Y = 0$ and $\pm 1$. The weak-interaction Lagrangian density in the presence of the $\phi$ field is given by
\begin{equation}
L_{\text{weak}} = G \sum (\Delta Y) \exp[-i \theta(\Delta Y) \phi].
\end{equation}
The Lagrangian density of the entire system can be written as
\begin{equation}
L = L_\phi + L_\psi + L_{\text{int}},
\end{equation}
in which the free-field Lagrangian density is given by $L_\phi$ and $L_\psi$, where
\begin{equation}
L_\phi = -\frac{1}{2} (\partial \phi/\partial x_\mu)^2,
\end{equation}
\begin{equation}
L_\psi = -\sum_a \gamma^\nu \gamma^\mu (\partial \psi_a/\partial x_\mu + m_a) \psi_a^\dagger,
\end{equation}
and the interaction Lagrangian density is given by
\begin{equation}
L_{\text{int}} = L_{\text{strong}} + L_{\text{elect}} + L_{\text{weak}},
\end{equation}
where $L_{\text{strong}}$ and $L_{\text{elect}}$ are, respectively, the Lagrangian density for the strong interaction and the electromagnetic interaction, and $L_{\text{weak}}$ is given by Eq. (18). For simplicity, we include only the free fermions in Eq. (21). All repeated indices are to be summed over.

The Lagrangian density $L$ is invariant under the Lorentz transformation, the time-reversal transformation, the $CP$ transformation, and the hypercharge gauge transformation. By using $L$, the equation of motion is found to be
\begin{equation}
(\partial^2 \phi/\partial x_\mu)^2 = - (\partial^2 \phi/\partial x_\mu) L_{\text{weak}}
\end{equation}
and
\begin{equation}
\gamma^\nu \left[ \frac{\partial \gamma^\nu}{\partial x_\mu} + m_a \right] \psi_a^\dagger = \frac{\partial}{\partial \phi_a} L_{\text{int}} - \frac{\partial}{\partial x_\mu} \left[ \frac{\partial}{\partial \psi_a^\dagger} L_{\text{int}} \right],
\end{equation}
where
\begin{equation}
\psi_{a, \mu} = (\partial/\partial x_\mu) \psi_a^\dagger.
\end{equation}
The hypercharge current density $J_\mu$ is related to the Lagrangian density by
\begin{equation}
J_\mu = -i \sum_a Y_a \left[ \frac{\partial L}{\partial \psi_a^\dagger} \psi_a - \psi_a^\dagger \frac{\partial L}{\partial \psi_a} \right] + \lambda^{-1} \frac{\partial L}{\partial \phi_a},
\end{equation}
where
\begin{equation}
\phi_{a, \mu} = \partial \phi/\partial x_\mu.
\end{equation}
From the equations of motion and the gauge-invariance property of $L$, the conservation law [Eq. (10)]
\begin{equation}
\partial J_\mu/\partial x_\mu = 0
\end{equation}
follows. An alternative form of \( J_\mu \) is given by Eq. (9)

\[
J_\mu = j_\mu - \lambda^{-1}(\partial \phi / \partial \sigma_\mu),
\]

where

\[
j_\mu = -i \sum_a Y_a \left[ \frac{\partial L}{\partial \psi_{\sigma a}} - \frac{\partial L}{\partial \psi_{\sigma a}^\dagger} \right].
\]  

The usual canonical formalism can be directly applied to the present problem. The conjugate momentum of the \( \phi \) field is given by

\[
\pi = \partial \phi / \partial \xi.
\]

The corresponding Hamiltonian density can be written as

\[
H = H_\phi + H_\psi + H_{\text{int}},
\]

where

\[
H_\psi = \frac{1}{2} \sum_{\alpha} \left[ \sum_{\sigma} \psi_{\sigma a} \left( \alpha - v + \beta a \right) \psi_{\sigma a}^\dagger \right],
\]

and

\[
H_{\text{int}} = H_{\text{strong}} + H_{\text{elect}} + H_{\text{weak}}.
\]

The matrices \( \alpha \) and \( \beta \) are the usual Dirac matrices. The \( H_{\text{strong}}, H_{\text{elect}} \) and \( H_{\text{weak}} \) describe, respectively, the strong, the electromagnetic, and the weak interaction which includes the modification due to the \( \phi \) field.

For simplicity, we have assumed in Eq. (30) that except for \( \phi \) all other fields are Fermion fields. To simplify further our discussions, we consider, in the following, only the case that \( L_{\text{int}} \) does not contain derivatives of \( \psi_a \). (All our conclusions can, of course, be applied to any other case.) Thus,

\[
H_{\text{weak}} = -L_{\text{weak}}
\]

and \( j_\mu \) is given by Eq. (11). Both \( \pi \) and \( \phi \) are Hermitian operators which satisfy

\[
[\pi(\tau,t),\phi(\tau',t)] = -i\hbar(\tau - \tau'),
\]

and the \( \psi_a \) obeys the usual anticommutation relation.

3. A CANONICAL TRANSFORMATION

To exhibit more explicitly the constant of motion implied by the conservation of hypercharge current, we consider the canonical transformation\(^4\)\(^1\) generated by the unitary operator

\[
U = \exp \left[ i \int \phi_0 \rho \right],
\]

where

\[
\rho = -i j_\mu = \sum_a Y_a \psi_a \psi_a^\dagger.
\]

and is the hypercharge density of the \( \psi_a \)'s. Under the canonical transformation,

\[
U \phi U^\dagger = \phi \exp (-i \lambda Y_\phi),
\]

\[
U \pi U^\dagger = -\pi - \lambda \rho.
\]

The Hamiltonian density is transformed into

\[
H_\phi = U H U^\dagger = H_\phi + H_\psi + H_{\text{strong}} + H_{\text{elect}} + H_{\text{weak}}(0)
\]

\[
-\lambda \left[ \rho \pi + \rho \cdot \nabla \phi \right] + \frac{1}{2} \lambda \rho^2,
\]

where \( j \) is the spatial part of the 4-vector \( j_\mu \), the terms \( H_\phi, H_\psi, H_{\text{strong}}, \) and \( H_{\text{elect}} \) are the same as that given by Eqs. (29)–(31), but \( H_{\text{weak}}(0) \) is related to Eq. (17) by

\[
H_{\text{weak}}(0) = -L_{\text{weak}}(0),
\]

and is independent of \( \phi \). In \( H_\phi \), the coupling between \( \phi \) and \( \psi_a \) is of a derivative type with \( \lambda \) as the coupling constant.

To understand the meaning of conservation of hypercharge in this new representation, it is convenient to consider the Fourier expansions of \( \phi \) and \( \pi \) in a volume \( \Omega \):

\[
\phi(\mathbf{r}) = \Omega^{-1/2} \sum_{k=0} \phi_k \exp (ik \cdot r),
\]

and

\[
\pi(\mathbf{r}) = \Omega^{-1/2} \sum_{k=0} \pi_k \exp (-ik \cdot r),
\]

where

\[
\pi_k = \pi_{-k}^\dagger, \quad \phi_k = \phi_{-k}^\dagger,
\]

and the commutation relations

\[
[\pi_k, \phi_{k'}] = -i \hbar \delta_{kk'}
\]

are satisfied for all \( k \) and \( k' \). The transformed Hamiltonian \( H_\phi \) is independent of \( \phi_k \). Thus,

\[
d\pi_k/dt = 0.
\]

The conservation of hypercharge, after the canonical transformation, becomes simply

\[
\pi_0 = \text{constant},
\]

which is a quantum number characterizing the particular state of the system. Under \( CP \) transformation, any state with \( \pi_0 \neq 0 \) is transformed into another state which has a \( \pi_0 \) of the opposite sign but with the same magnitude. The \( \pi_0 \) is related to the \( Q \), introduced in Eq. (12), by \( \pi_0 = \Omega^{-1/2}(\lambda Q) \).

It is important to notice that the hypercharge gauge transformation becomes a totally trivial operation after the canonical transformation. This may at first sight seem rather strange, but it is actually a general feature of such gauge invariance. In the Appendix, we give a simple example to illustrate further the same property.

4. APPARENT \( CP \) NONINVARIANCE

In this section we discuss the various consequences for the state which has a \( \pi_0 \neq 0 \). From Eq. (39), we
notice that, after the canonical transformation, the Hamiltonian can be written as
\[
H = T + V \int \rho d^3r + \int H' d^3r,
\]
where
\[
H' = H_0 + H_N + H_{\text{strong}} + H_{\text{elect}} + H_{\text{weak}}(0) + H_1
\]
and
\[
H_1 = -\lambda [\rho \sigma + \frac{1}{2} \nabla \rho \sigma] + \frac{1}{2} \lambda \rho^2.
\]
In Eq. (47), the operators \( \rho \sigma \) and \( \pi \) are defined to be
\[
\rho = \sum_{k<0} \rho_k \exp(ik \cdot r)
\]
and
\[
\pi = \sum_{k>0} \rho_k \exp(-ik \cdot r).
\]
The first term on the right-hand side of Eq. (45) shows that there is an energy difference \( V \) between any hypercharge particle and its antiparticle of the same momentum. For particles with \( Y = 1 \), this potential energy difference is given by
\[
V = -\frac{1}{2} \frac{\gamma V^0 (\pi^0 - m^0 \gamma^0)}{m^0}
\]
As mentioned in the Introduction, the existence of such an energy difference implies that \( K^{\pm} \rightarrow 2\pi \) can occur and the analysis made in Ref. 2 is applicable. In the following, we list the various consequences for the weak decays. Most of these results have already been stated in Ref. 2. (i) In a vacuum (i.e., in the absence of any neighboring matter), the two states \( |K^0\rangle \) and \( |K^0\rangle \) of which has a single lifetime, are related to \( |K^0\rangle \) and its \( CP \) conjugate state \( |\bar{K}^0\rangle \) by
\[
|K^0\rangle = \frac{1}{\sqrt{2}} \left( |\bar{K}^0\rangle + |K^0\rangle \right)
\]
and
\[
|K^0\rangle = \frac{1}{\sqrt{2}} \left( |\bar{K}^0\rangle - |K^0\rangle \right)
\]
where \( |K^0\rangle \) and \( |\bar{K}^0\rangle \) are time-independent,
\[
\lambda_1 = \lambda + \frac{1}{2} \gamma (\bar{m}^2 - m^2),
\]
\[
\lambda_2 = \lambda - \frac{1}{2} \gamma (\bar{m}^2 - m^2),
\]
and
\[
\eta = (2\gamma)^{-1} \left[ (m^2 - \bar{m}^2) - i \frac{1}{2} \left( \Gamma^0 + \Gamma^0 \right) \right],
\]
\[
\xi = (2\gamma)^{-1} \left[ (m^2 - \bar{m}^2) - i \frac{1}{2} \left( \Gamma^0 - \Gamma^0 \right) \right],
\]
where \( \gamma \) is given by Eq. (16). Since \( V \neq 0 \), the actual mass and width of \( K^0 \) and \( K^0 \) are determined by \( \lambda_1 \) and \( \lambda_2 \). We may define
\[
\lambda_{j1} = \gamma \left[ m_j - i \frac{1}{2} \Gamma_j \right],
\]
where \( j = 1, 2 \). The “observed mass” \( m_1, m_2 \) and the “observed width” \( \Gamma_1, \Gamma_2 \) depend on \( \gamma \). If \( |\epsilon| < \epsilon^2 \) (which corresponds to \( \gamma < 10^3 \)), then
\[
(m_1 - i \frac{1}{2} \Gamma_1) \approx (m^0 - i \frac{1}{2} \Gamma^0) + \frac{\gamma V}{\Delta m^0 - i \Delta \Gamma^0} \]
and
\[
(m_2 - i \frac{1}{2} \Gamma_2) \approx (m^0 - i \frac{1}{2} \Gamma^0) - \frac{\gamma V}{\Delta m^0 - i \Delta \Gamma^0}.
\]
For large values of \( \gamma (\gg 10^3) \),
\[
\lambda \approx - \frac{1}{2} \gamma \left[ (m^0 + m^0) - i \frac{1}{2} \left( \Gamma^0 + \Gamma^0 \right) \right],
\]
\[
\lambda \approx - \frac{1}{2} \gamma \left[ (m^0 + m^0) - i \frac{1}{2} \left( \Gamma^0 + \Gamma^0 \right) \right].
\]
Thus, the lifetimes of \( K^0 \) and \( K^0 \) become the same at the extremely high-energy limit.

(iii) From Eqs. (50), (51), and the \( CP \) invariance property of the theory, we find (in vacuum)
\[
\frac{\text{Rate}(K^0 \rightarrow \pi^+ + \pi^-)}{\text{Rate}(K^0 \rightarrow \pi^+ + \pi^-)} = \frac{|\epsilon|^2}{|\epsilon|^2}.
\]
For \( \gamma < 10^3 \), the parameter \( |\epsilon|^2 \) varies according to \( \gamma^2 \); for \( \gamma > 10^3 \), the parameter \( |\epsilon|^2 \) is fixed.

For the \( 3\pi \) decay mode, if we neglect the high angular momentum states in the final \( (\pi^- + \pi^+ + \pi^0) \) system, then
\[
\frac{\text{Rate}(K^0 \rightarrow \pi^- + \pi^+ + \pi^-)}{\text{Rate}(K^0 \rightarrow \pi^- + \pi^-)} = \frac{\text{Rate}(K^0 \rightarrow \pi^+ + \pi^- + \pi^0)}{\text{Rate}(K^0 \rightarrow \pi^+ + \pi^- + \pi^0)} = |\epsilon|^2
\]
If we assume the \( \Delta Q = \Delta S \) rule for the leptonic decay modes, then
\[
\frac{\text{Rate}(K^0 \rightarrow \pi^- + l^- + \nu_l)}{\text{Rate}(K^0 \rightarrow \pi^+ + l^- + \nu_l)} = \frac{\text{Rate}(K^0 \rightarrow \pi^- + l^- + \nu_l)}{\text{Rate}(K^0 \rightarrow \pi^+ + l^- + \nu_l)} = \frac{\text{Rate}(K^0 \rightarrow \pi^+ + l^- + \nu_l)}{\text{Rate}(K^0 \rightarrow \pi^+ + l^- + \nu_l)} = |\epsilon|^2
\]
and
\[
\frac{\text{Rate}(K^0 \rightarrow \pi^+ + \mu^+ + \nu_\mu)}{\text{Rate}(K^0 \rightarrow \pi^+ + \mu^- + \bar{\nu}_\mu)} = \frac{1+\epsilon^2}{1-\epsilon},
\]  
(70)

where \(\epsilon = e \mu\).

(iii) All above results [Eqs. (50)–(54) and (65)–(70)] are applicable to \(K^0\) and \(\bar{K}^0\) in a medium, provided we replace Eqs. (55)–(57) by
\[
\eta = (2\pi)^{-\frac{3}{2}} \left[ (m_{\pi^0}^2 + m_{\pi^0}^2) - i\frac{1}{2} (\Gamma_{\pi^0} + \Gamma_{\pi^0}) \right],
\]
\[
(71)
\frac{1}{2} (n + n') \alpha \kappa v,
\]
and
\[
\xi = (2\pi)^{-\frac{3}{2}} \left[ (m_{\pi^0}^2 - m_{\pi^0}^2) - i\frac{1}{2} (\Gamma_{\pi^0} - \Gamma_{\pi^0}) \right],
\]
\[
(72)
\frac{1}{2} (\sqrt{n^2 + n'^2} \alpha \kappa v),
\]
where \(k\) is the momentum of the \(K\) meson and \(n, n'\) are, respectively, the complex index of refraction of \(K^0\) and \(\bar{K}^0\) in the medium.

The sign of \(V\) can be determined either by observing the interference term between the decay of \(K^0\) with that of \(\bar{K}^0\) in vacuum, or by studying the decay rates of \(K^0\) or \(\bar{K}^0\) in a medium.

In all above formulas, the \(\gamma\) is measured with respect to the rest system of the entire volume \(\Omega\). On the other hand, we can also measure the value of \(\gamma\) with respect to the earth by direct means. Combining these two measurements, it would become possible to measure the "absolute" velocity of the earth.

(iv) In the decays of all other particles, no apparent \(CP\) noninvariant term can be observed if an energy difference of the order of \(10^{-8}\) eV between the particle and its antiparticle can be neglected.

5. EMISSION AND ABSORPTION OF \(\phi\) QUANTA

From Eq. (47) we find that the coupling between \(\phi\) and \(\psi\) is of a derivative type and that the coupling parameter, \(\lambda\) has the dimension of a length. It is convenient to introduce a dimensionless coupling constant, \(f\):
\[
f = \lambda m_N,
\]  
(74)

where \(m_N\) = mass of the nucleon. To study the emission and absorption processes of \(\phi\) quanta with \(k=0\), we briefly describe the rules for Feynman graphs.

Let us start with \(H_V\) [Eq. (45)] and use the interaction representation. From Eq. (48), it follows that as \(\Omega \rightarrow \infty\), the propagator of \(\phi_1\) is given by
\[
\phi_1(x)\phi_1(y) = (16\pi^4)^{-\frac{1}{2}} \int (ik^4)^{-\frac{1}{2}} \exp[i k_\mu(x-y)_\mu]d^4k,
\]
\[
= \frac{1}{2} D_\phi(x-y),
\]  
(75)

where \(k^2 = k_\mu k^\mu\) and \(d^4k\) is real. It is well known that the second time derivative of the right-hand side of Eq. (75) differs from the corresponding contraction of the time derivatives of \(\phi_1\) by a \(\delta^4(x-y)\) function which, in the evaluation of the \(S\) matrix, contributes terms that are exactly canceled by the corresponding terms generated by the \(\frac{1}{2} m_p^2\) in \(H_1\) [Eq. (47)]. Therefore, in deriving the Feynman rules we can ignore the \(\delta^4(x-y)\) function, but regard the contraction of the derivatives of the \(\phi_1\) as given by
\[
\left[ \frac{\partial}{\partial x_\mu} \phi_1(x) \right] \left[ \frac{\partial}{\partial y_\nu} \phi_1(y) \right] = \frac{\partial^2}{\partial x_\mu \partial x_\nu} \left[ \frac{1}{2} D_\phi(x-y) \right],
\]  
(76)

and, at the same time, replace Eq. (47) by
\[
H_1 = -\int \frac{f^2}{m_N} \frac{\partial}{\partial x_\mu} \phi_1(x).
\]  
(77)

In the limit that the weak-coupling constant \(G\rightarrow 0\), the current \(j_\mu\) is conserved. From Eq. (77), it follows that the emission and absorption rate of any \(\phi\) quanta with \(k=0\) must be zero if \(G=0\). Therefore, the actual emission and absorption probability for each \(\phi\) quantum is proportional to \(f^2 m_N^2\). The same conclusion can also be derived in a more transparent way by using the original Hamiltonian, \(H\) given by Eq. (28).

The \(\phi\) quanta can be emitted whenever there is a violation of the conservation of \(j_\mu\) such as, e.g.,
\[
\Lambda^0 \rightarrow \rho + \pi^-.
\]  
(78)

The rate for the process
\[
\Lambda^0 \rightarrow \rho + \pi^- + \phi
\]  
(79)
is (to lowest order in \(G^2\) and \(f^2\) proportional to
\[
f^2 |M|^2 (16\pi^4 E_{\phi} m_N^2)^{-\frac{1}{2}} d^4k,
\]  
(80)

where \(M\) is the matrix element for the \(\Lambda^0 \rightarrow \rho + \pi^-\) vertex in the \(\phi\) emission, \(k\) is the momentum of \(\phi\) and \(E_{\phi} = |k|\). For the emission of soft \(\phi\) quantum, we can regard \(M\) as independent of \(k\). The branching ratio is given by
\[
\frac{\text{Rate}(\Lambda^0 \rightarrow \rho + \pi^- + \phi)}{\text{Rate}(\Lambda^0 \rightarrow \rho + \pi^-)} = f^2 E_{\phi} (8\pi^2 m_N^2)^{-\frac{1}{2}},
\]  
(81)

where \(E_{\phi}\) is the maximum energy of the soft \(\phi\) quantum emitted. Similarly, the rate for
\[
\Lambda^0 \rightarrow \rho + \pi^- + \phi_1 + \phi_2 + \cdots + \phi_N
\]  
(82)
is (to lowest order in \(G^2\) and \(f^2\) proportional to
\[
(n!)^{-\frac{1}{2}} f^2 N |M|^2 \prod_i (16\pi^4 E_{\phi} m_N^2)^{-\frac{1}{2}} d^4k_i,
\]  
(83)

where \(k_i\) is the momentum of \(\phi_i\) and \(E_i = |k_i|\). For soft
\footnote{See G. C. Wick, Phys. Rev. 80, 268 (1950) for the definition of \(\phi(x)\phi(y)\).}
\footnote{For a detailed proof of this well-known result see, for example, the Appendix in T. D. Lee and C. N. Yang, Phys. Rev. 128, 885 (1962).}
quanta emission, the corresponding branching ratio

\[ B_n = \frac{\text{Rate}(\lambda^0 \rightarrow \rho + \pi^+ + n\phi)}{\text{Rate}(\lambda^0 \rightarrow \rho + \pi^-)} \]  

is given by

\[ B_n = \left[ (2\pi)^n \ln \gamma \right]^{-1} \left[ \frac{f^2 E_n^2 (4\pi^2 m_N^2)^{-1}}{n} \right], \]

where \( E_n \) is the maximum value of the total energy given to the \( n \) \( \phi \)-quanta system.

Identical formulas can be applied to the \( \phi \)-emission probability associated with any \( \Delta Y \neq 0 \) weak reactions. From existing data, we know that

\[ f \ll 1. \]

Indeed, it seems that \((4\pi)^{-1}f^2\) can be as big as the fine structure constant without violating any known observations.

For \( \phi \) quantum of sufficiently high energy, reactions such as \( \phi^+ p \rightarrow \Lambda^0 + \pi^+ \) can occur. The corresponding cross section is expected to be \( \sim G_f^2 \). If the energy of \( \phi \) is \( \lesssim 175 \) MeV, then to the lowest order of \( G_f^2 \) (but to arbitrary orders in \( f^2 \) and \( e^2 \)) the \( \phi \) quantum cannot be absorbed by a nucleon at rest. Thus, the mean free path of \( \phi \) in a medium is, in general, much longer than that of the neutrino.

So far, we consider the case that \( \phi \) is related to the hypercharge gauge transformation (or strangeness-gauge transformation). Therefore, for weak decays which satisfy \( \Delta Y = 0 \), the emission probability for \( \phi \) is zero. On the other hand, if \( \phi \) is connected with the gauge transformation associated with the isospin component of isotopic spin, \( I_n \), then reactions such as

\[ n \rightarrow \rho + e^- + \nu_e + \phi \]  

or

\[ \phi + p \rightarrow n + l^+ + \nu_l \]

become possible \((l = \epsilon \text{ or } \mu)\).

6. REMARKS

(i). The possibility that our system (or universe) is in a state with \( \pi_1 \neq 0 \) evokes many questions concerning its nature. From Eqs. (49), (66), and (74), we notice that the energy density due to \( \pi_1 \) is given by

\[ \frac{4}{3} \Omega^{-1} \pi_1 \rho = (8\pi^2)^{-1} V^7 m_N \geq 10^7 \text{ BeV/cm}^3. \]

The total energy in the \( k = 0 \) mode is proportional to the entire volume \( \Omega \) of the system (which is much greater than the rest-mass energy of the baryons in the universe). The corresponding energy contained in any other mode in the ground state of the system is the well-known zero-point energy \( \frac{1}{2} |k| \). The zero-point energy density contained in all the \( k \neq 0 \) modes diverges in the ultraviolet region. If we introduce a momentum cutoff \( k_{\text{max}} \gg m_N \), the energy density in all the \( k \neq 0 \) states is

\[ (8\pi^2)^{-1} k_{\text{max}} \gg 10^{34} \left( \frac{4}{3} \Omega^{-1} \pi_1 \rho \right). \]

Although the fractional energy contained in the \( k = 0 \) mode is extremely small, the fact that its average value is of a macroscopic nature gives rise to the many striking physical effects discussed in this paper.

The absorption and scattering of the \( \phi \) quantum in matter have some similarities with neutrinos. Thus, it is of some interest to compare the energy density contained in the \( k = 0 \) mode of the \( \phi \) field with the degenerate neutrino energy density of the universe, which is, of course, not known. If we make the arbitrary assumption that these two energy densities should be comparable, then the Fermi momentum of the degenerate neutrinos would be \( \sim 10^7 \) eV. As has been discussed by Weinberg,\(^7\) such a high value of Fermi momentum is, theoretically, not impossible. Similarly, an energy density of the magnitude given by Eq. (89) may also be possible for the \( \phi \) field in some oscillatory cosmological models.

(ii). The present theory resembles the Bose-Einstein condensation of a system of Bose particles. In either case, the \( k = 0 \) mode acquires a macroscopic value for its occupation number. Such solutions are intrinsically different in both mathematical content and physical manifestation from the usual ones. It is possible that the existence of such eigensolutions is a general feature of any boson system. In this sense, the present theory may also serve as a simple model for a general class of field theories.

In the case of Bose gas, it is known that there are quasistationary states which correspond to the flow of superfluid. The simplest way is to divide the entire volume into many smaller but macroscopic boxes, and to allow variations between different boxes. The same can also be done in the present case by allowing the value for \( \pi_1 \) to change gradually from one box to the next one. In this way, we can construct a macroscopic field \( \pi_1(\epsilon) \). The solution \( \pi_1(\epsilon) \neq 0 \) is not an eigenstate of the Hamiltonian of the entire system, but it might be regarded as a quasistationary solution. Such a macroscopic field, if it exists, should have some cosmological influences, and the present value of \( \pi_1 \) as determined by the observed \( K^0 \rightarrow 2\pi \) rate may, then, fit into a general picture concerning the evolution of our universe.

(iii). An entirely different mechanism has been suggested,\(^8\) to account for the \( 2\pi \) decay mode of \( K^0 \) and, at the same time, to maintain \( CP \) invariance by introducing a long-range spin-1 field \( V_\rho \) interacting with the hypercharge (or \( I_\rho \)) current density \( j_\rho \) of the presently known particles.

Since \( j_\rho \) is not conserved, \( V_\rho \) must have a mass \( m \).

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\(^7\) S. Weinberg, Phys. Rev. 128, 1457 (1962).


(There is an unpublished note by F. Gursey and A. Pais in which they considered the possible existence of a pseudoscalar field in connection with the experiment by Christeren et al. The proposal by Gursey and Pais seems to be totally different from the theory discussed in this paper and from the suggestion made by Bernstein, Cabibbo and Lee, and by Bell and Perring.)
It has been pointed out by Weinberg$^9$ that the mass, $m$, cannot be too small. In order to account for the observed rate of $K^0 \rightarrow 2\pi$, the experimental absence of real emission of such $V_\nu$ quanta and the present experimental accuracy$^5$ of the equality between the observed gravitational mass and the inertial mass, the Compton wavelength $\hbar/c$ of $V_\nu$ should be about (or less than) the radius of the earth. The corresponding (coupling constant)$^2$ is about (or larger than) $10^{-46}$. Thus, such a field, if it exists, could also be detected by improving the present accuracy of the Čečovs-type experiment by another order of magnitude. In contrast, the theory discussed in this paper does not give any such observable effect in the Čečovs-type experiment.

If the vector field $V_\nu$ exists, its predictions on all apparent $CP$ noninvariant phenomena are the same as that discussed in the present paper. These two different theories can be differentiated either by improving the present accuracy on the equality between the gravitational mass and inertial mass or by observing the real emission of either the $V_\nu$ or the $\phi$ quantum and determining its spin.

Another variation on the same theoretical idea is to couple $V_\nu$ with $J_\mu$ [Eq. (9)]. Since $J_\mu$ is conserved, $V_\nu$ could then also have zero mass. While such a model has some appeal because of its possible connection with the gauge invariance of the second kind, it does appear at the present time to be too speculative to warrant a full discussion.

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APPENDIX

In this Appendix we give a simple example of the interaction between an electron and the phonon field in a solid to illustrate the interplay between the gauge transformation, the canonical transformation, and the apparent asymmetry phenomenon.$^4$ Let the Hamiltonian be given by

$$ H = \frac{\hbar^2}{2m} \sum \omega_k (\alpha_k^* \alpha_k + \frac{1}{2}) $$
$$ + \sum [V_\nu \alpha_k e^{ikr} + V_\nu^* \alpha_k^* e^{-ikr}], \quad (A1) $$

where $r, p$ are the coordinate and momentum of the electron, $\alpha_k$ and $\alpha_k^*$ are the annihilation and creation operators of the phonon, $\omega_k$ is its frequency and $V_\nu$ is the interaction form factor. The Hamiltonian (A1) is invariant under the transformation

$$ \alpha_k \rightarrow \alpha_k e^{i\kappa \cdot d}, $$
$$ \alpha_k^* \rightarrow \alpha_k^* e^{-i\kappa \cdot d} $$

and

$$ r \rightarrow r - d. \quad (A2) $$

This invariance is connected with the conservation law that the total-momentum operator

$$ p + \sum \alpha_k^* \alpha_k k $$

commutes with $H$.

We may introduce a canonical transformation generated by a unitary matrix

$$ U = \exp \{i \sum \alpha_k^* \alpha_k k \cdot r \}, $$

which transforms

$$ U \alpha U^\dagger = \alpha_k e^{-i\kappa r}, $$
$$ U r U^\dagger = r, $$

and

$$ U p U^\dagger = p - \sum \alpha_k^* \alpha_k k. \quad (A5) $$

Thus, the transformed Hamiltonian becomes

$$ U H U^\dagger = \frac{1}{2m} (p - \sum \alpha_k^* \alpha_k k)^2 + \sum \omega_k (\alpha_k^* \alpha_k + \frac{1}{2}) $$
$$ + \sum [V_\nu \alpha_k + V_\nu^* \alpha_k^*]. \quad (A6) $$

In the transformed system, $U H U^\dagger$ is independent of $r$; therefore $\dot{p} = 0$, and the conservation of momentum becomes simply

$$ p = \text{constant}. \quad (A7) $$

Furthermore, after the canonical transformation, the gauge transformation (A2) becomes an identity transformation for the transformed $\alpha_k$.

This simple example shares all the features of the theory discussed in the paper. Indeed, if the system (after the canonical transformation) is in a state with

$$ p \neq 0, $$

the first term on the right-hand side of (A6) contains a part which is of the form

$$ \frac{1}{m} \sum (p \cdot k) \alpha_k^* \alpha_k. \quad (A8) $$

This implies that the energy of a phonon with momentum $k$ is different from that of $-k$.

We may carry the analogy further and imagine that for some observational reasons it is easy to detect the phonons, but the existence of the extra electron is not known. Thus, it would appear that in this solid phonons are created and annihilated spontaneously and there is an apparent nonconservation of momentum. By postulating the existence of an additional electron it is possible to construct the Hamiltonian, Eq. (A1), which is invariant under the gauge transformation (A2), thus conserving momentum, and which is invariant under the discrete space inversion transformation $k \rightarrow -k$. However, if the system is in a state with a nonzero value of total momentum, there would exist an apparent noninvariance under space inversion as well, which is in complete analogy with the apparent $CP$ noninvariance discussed in the paper.

Analysis of CP-Noninvariant Interactions and the $K_0^0$, $K_2^0$ System*

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In order to explain the recent observation of $K_2^0 \rightarrow \pi^+ + \pi^-$ by Christenson et al., we assume that the relevant interaction Hamiltonian can be separated into two parts: $H_0 + H_F$, where $H_0$ is the usual CP-conserving and $T$-invariant weak interaction and $H_F$ is a new interaction which is not invariant under $CP$. Depending on the selection rule of $H_F$ with respect to the strangeness quantum number $S$, this new interaction $H_F$ can be much stronger than $H_0$, or somewhat weaker than $H_0$, or very much weaker than $H_0$. The properties of these different classes of possibilities are discussed and some possible experimental tests are suggested. The system of $K_2^0$ and $K_0^0$ is analyzed for these different possibilities without assuming the validity of $CPT$ invariance. Efforts are made to separate out the various experimental consequences that follow from different symmetry requirements which may be satisfied by this new CP-noninvariant interaction $H_F$.

I. GENERAL DISCUSSIONS

RECENTLY, Christenson et al.1 observed that the long-lived component of the $K_0^0$, the $K_2^0$, decays into two $\pi$ mesons with a rate given by

$$|\epsilon| = \frac{\text{Rate}(K_2^0 \rightarrow \pi^+ + \pi^-) / \text{Rate}(K_0^0 \rightarrow \pi^+ + \pi^-)}{\approx 2.2 \times 10^{-4}}. \tag{1}$$

Throughout this paper we assume that these reactions can occur in a vacuum which has a definite $CP = +1$ value.2 The operators $C$ and $P$ denote, respectively, the charge conjugation and the space inversion which are defined by the strong and the electromagnetic interactions. Thus, the final state $(\pi^+ + \pi^-)$ must have a definite $CP = +1$ value. Since out of the states $K_0^0$ and $K_2^0$ only one initial state with $CP = +1$ can be constructed, the observation that both $K_2^0$ and $K_0^0$ can decay into $(\pi^+ + \pi^-)$ shows that $CP$ cannot be conserved in the $K_2^0$ decay. If $CPT$ invariance is assumed, then the time-reversal invariance must also be violated in the $K_2^0$ decay. From the magnitude of the observed value $|\epsilon|$ and the already existing experimental limits on the possible $CP$ violation in other weak processes,3 it appears that the amount of either $CP$ violation or the

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time-reversal violation, at least in these particular reactions, is relatively small.

We assume that it is possible to separate the relevant interaction Hamiltonian into two parts:

$$H_0 + H_F, \tag{2}$$

where $H_0$ is the usual weak interaction which satisfies the $\Delta S = \pm 2$ rule and is invariant under $CP$ and $T$ separately, where $S$ is the strangeness quantum number and $T$ is the time reversal operator. The remainder $H_F$ is a new interaction which does not conserve $CP$, nor is it invariant under $T$. For order-of-magnitude estimations, we may regard $H_0$ and $H_F$ as characterized, respectively, by the coupling constants $G$ and $F$ where $G$ can be either the Fermi coupling constant $G_F$ in $\beta$ decay, or the usual strangeness-nonconserving weak coupling constant $\sim 1/2G_F$, and $F$ is a new coupling constant which has the same dimension as $G_F$.

Clearly, with only one observed $CP$-violating reaction, a great variety of different CP-noninvariant Hamiltonian $H_F$ can be proposed. It is important to notice that, at present, not only is the detailed form of $H_F$ unknown, but even the approximate order of magnitude of its coupling constant $F$ is limited only within an extremely wide range by the existing experiments. As we shall see, there are three very different classes of possibilities depending on the particular selection rule that $H_F$ satisfies with respect to the strangeness quantum number $S$. These three classes are listed in Table I. Some specific properties of these three classes of possibilities will be discussed in the following:

(1) If $H_F$ satisfies the $\Delta S = 0$ rule and if it does not contain any leptonic operators, then $(F/G)$ can be $\approx 10^9$. Thus, $H_F$ is much stronger than the usual weak interaction $H_0$. The dimensionless coupling constant of $H_F$ is given by (within a factor $\sim 10$)

$$(4\pi)^{-1} Fm_p^2 \approx |\epsilon| \approx 2.2 \times 10^{-4}, \tag{3}$$

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where $m_p$ is the mass of the proton. Assuming that CPT invariance holds, the observation of the $(\pi^+\pi^-)$ decay mode of $K^0\bar{K}^0$ and the present limit on the electric dipole moment of the neutron require that $H_F$ must conserve $P$ but violate $C$ and $T$ invariance. The question whether $H_F$ may or may not contain terms that violate the conservation of isotopic spin $I$ can be left open. We note that if $H_F$ satisfies the $|\Delta I|=0$ rule, then it is convenient to regard $H_F$ as a small $C$-, or $T$-, noninvariant part of the usual strong interaction; otherwise, it is more convenient to regard $H_F$ as a completely new interaction independent of the existing strong interactions.

The decay of $K^0\bar{K}^0$ to $2\pi$, as well as other $CP$, or $T$, violating weak processes, can occur through the second-order expression $H_2H_F$, which contains terms that violate the strangeness conservation, $P$ invariance, $C$ invariance, $PC$ invariance, and $T$ invariance. The possible existence of such a $H_F$ can also be tested by examining the limits of $T$ invariance and $C$ invariance in the strong interactions. If Eq. (5) holds, then all nuclear matrix elements could have a small admixture of $T$-noninvariant amplitude which is $|\Delta S|=1$ times the $T$ invariant amplitude. Such a $T$-noninvariant amplitude can be observed by studying, e.g., the E2, M1 interference terms in an appropriate nuclear transition.

Another possible test of such a relativistically strong $H_F$ was pointed out by Friedberg, Lee, and Schwartz. The decay of $\eta$ is known to violate the $G$ parity. In particular, the decay mode

$$\eta \rightarrow 3\pi$$

(4)

can occur via the virtual electromagnetic interactions to a final $(3\pi)$ state with $I=1$ and $C=-1$. It can also decay, now, through $H_F$ to a final state with $C=-1$ and $I=0$, or 2, with a relative amplitude $\sim r$ of $r$ of $O(1)$, where $r$ is the fine structure constant and $r$ is a reduction factor depending on the final-state pion interactions and the angular momentum barrier effects. The approximate order of magnitude of $r$ is expected to be $\leq 1$, if $H_F$ contains a $|\Delta I|=2$ part; otherwise, it could be much smaller than 1. The interference between the $C=-1$ amplitudes can result in an asymmetry between $\pi^+$ and $\pi^-$ in the Dalitz plot of the final $(\pi^+,\pi^-,\pi^0)$ distribution. Because of the known strong pion interactions in the final states, such an asymmetry does not violate $CP$ invariance. The detection of such an asymmetry would be an unequivocal proof of violation of $C$ invariance in the $\eta$ decay; it would also establish that the $C$-noninvariant interaction $H_F$ is much stronger than the usual weak interaction $H_G$.

(2) If $H_F$ allows the $\Delta S=\pm 1$ transitions, but forbids the $\Delta S=\pm 2$ transitions, then $F$ must be of the same order of magnitude as, or somewhat smaller than, $G$. (The $H_F$ may contain, in addition, also some $\Delta S=0$ transition amplitudes.) If the observed slow rate of $K^0\rightarrow \pi^+\pi^-$ is regarded as a typical example of $CP$ noninvariance, then $F$ should be smaller than $G$ and, within a factor $\sim 10$, is given by

$$F \approx 10^{-2}G$$

(5)

Such a $H_F$ can be easily described by assigning some appropriate phase factors to the various coupling constants in the usual weak interactions. The general observable effects in such a case have already been discussed in the literature.

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**Table I. Three different classes of CP-noninvariant interaction $H_F$.**

<table>
<thead>
<tr>
<th>$F/G$ (within a factor $\sim 10$)</th>
<th>Conditions satisfied by $H_F$</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sim 10^9$</td>
<td>$\Delta S=0$ transitions only $P$ is conserved $C$ invariance violated $T$ invariance violated</td>
<td>Both strong and weak processes have $C$-noninvariant and $T$-noninvariant amplitudes which are $\sim</td>
</tr>
<tr>
<td>$\sim 10^{-2}, 10^{-3}$</td>
<td>$\Delta S=\pm 1$ transitions allowed $\Delta S=\pm 2$ transitions forbidden $CP$ invariance violated</td>
<td>Only weak processes have $CP$-violating amplitudes.</td>
</tr>
<tr>
<td>$\sim 10^{-4}$</td>
<td>$\Delta S=\pm 2$ transitions allowed $CP$ invariance violated</td>
<td>Neglecting terms of $O(10^{-9})$, but keeping those of $O(1)$, only $K_\pi$, $\pi$ decays violate $CP$ invariance. These violations are due entirely to the mass operator of the $K_\pi$ system.</td>
</tr>
</tbody>
</table>

---


*If $H_F$ is not invariant under $CPT$, then there would be a mass difference between any strongly interacting particle and its antiparticle; e.g., $(K^0[H]|H[\bar{K}])$ could be different from $(\bar{K}^0[H]|H[\bar{K}])$. Such a difference is clearly ruled out if $F$ is $\sim 10^9$. |


with the observation of Christenson et al., many specific models have also been proposed. In this paper, we will not investigate further this well-studied case.

(3) The final possibility is that \( H_{\theta} \) may be very much weaker than \( H_{\beta} \); i.e.,

\[
P \approx 10^{-8} G
\]

or

\[
P \approx 10^{-12} m_p e^{-2}.
\]

In this case, \( H_F \) must allow a \( \Delta S = \pm 2 \) transition and, in particular, the matrix element

\[
\langle K_0 | H_F | K_0 \rangle \neq 0.
\]

It will be shown that, in this case, to the accuracy of keeping \( |4\pi F/G| m_p z^2 | \approx |\epsilon| \), but neglecting \( (F/G) \sim 10^{-9} \) as compared to 1, the \( CP \)-violating effects in all known weak processes are already completely determined, provided the validity of the \( CPT \) theorem is assumed; otherwise, an additional real parameter is needed.

We note that if one neglects \( F/G \sim 10^{-9} \), then in all known weak processes, except those involving \( K^0 \) and \( K_{\bar{0}} \), there is no observable \( CP \)-violating effect. For the \( K^0 \) and \( K_{\bar{0}} \) states, the order of \( F/G \) can occur, but only through the mass operator. Assuming the validity of the \( CPT \) theorem, the eigenstates \( K^0 \) and \( K_{\bar{0}} \), each of which has a definite lifetime, are given by

\[
K^0 = (1 + | \epsilon |^2)^{-1/2} [(1 + \epsilon) K^+ + (1 - \epsilon) K^-]
\]

and

\[
K_{\bar{0}} = (1 + | \epsilon |^2)^{-1/2} [(1 + \epsilon) K^- - (1 - \epsilon) K^+],
\]

where \( \epsilon \) is a complex parameter,

\[
\epsilon = | \epsilon | e^{i \theta}.
\]

The magnitude of \( \epsilon \) is proportional to \( F/G \). As will be shown in Sec. III, to the accuracy of neglecting \( F/G \), the parameter \( \epsilon \) has already been measured. The absolute magnitude of \( \epsilon \) is given by Eq. (1), and the phase \( \delta \) is given by

\[
tan \delta = -2 [(m_1 - m_2) / (\gamma_1 - \gamma_2)],
\]

where \( m_i \) and \( \gamma_i \) are the observed mass and width of \( K^0_i \) (i = 1 or 2). To the same accuracy, this parameter \( \epsilon \) determines all the \( CP \) violating effects in \( K_{\pm 1} \) decays.

Within the present experimental techniques, while it is possible to measure weak-interaction amplitudes to the accuracy of \( | | e | 2 \times 10^{-4} \), it seems rather unlikely that any known weak-interaction amplitude can be determined to the accuracy of \( F/G \sim 10^{-9} \). Thus, in this case the \( H_F \) is very much weaker than \( H_{\beta} \), we may consider a simple \( \text{phenomenological model} \), in which, except for the mass operator of the \( K^0, K_{\bar{0}} \) system, all matrix elements are assumed to conserve \( CP \) and to be invariant under \( T \). All \( CP \)-violating phenomena in this model are due to the \( CP \) noninvariance of the mass operator of the \( K^0 \) and \( K_{\bar{0}} \) states. The further assumption of \( CPT \) invariance requires all these \( CP \) violations to be described by one single phenomenological parameter \( \epsilon \), which has already been measured [except that Eq. (12) has two discrete solutions \( \delta \) and \( \delta + \pi/2 \)]. If the underlying \( CP \)-violating interaction \( H_F \) does satisfy the general conditions given by Eqs. (7) and (8), then to the assumed accuracy of neglecting \( F/G \) but keeping \( F/G \), all observable effects of such an extremely weak interaction are identical with this simple phenomenological model. If \( CPT \) invariance is violated, an additional real parameter \( \eta \) is needed to characterize the mass operator. To the same assumed accuracy, all observable \( CP \)-violating or \( CPT \)-violating effects of \( H_F \) contain only this single unknown parameter \( \eta \).

In the above, we summarize the three different possibilities of the \( CP \)-noninvariant interaction \( H_F \). The remaining part of the paper concerns, mainly, the \( K_{\pm 1} \) system. In Sec. II, we review the general properties of the \( K_{\pm 1} \) system, which are valid for all three classes of \( H_F \), and without assuming the validity of the \( CPT \) theorem. Applications to the simple phenomenological model, in which only the mass operator violates \( CP \) invariance, are given in Sec. III, if \( CPT \) invariance holds, and in Sec. IV, if \( CPT \) invariance is also violated.

II. GENERAL DESCRIPTION OF \( K^0 \) AND \( K_{\bar{0}} \) STATES

In this section, we review the general description of the \( K^0 \) and \( K_{\bar{0}} \) system which is valid for any interaction Hamiltonian and \textit{without} the assumption of \( CPT \) invariance. We consider a coherent mixture of \( K^0 \) and \( \bar{K}^0 \) whose amplitude at time \( t \) is described by the wave function

\[
\psi(t) = a(t) | K^0 \rangle + b(t) | \bar{K}^0 \rangle
\]

or simply as

\[
\psi(t) = \begin{pmatrix} a(t) \\ b(t) \end{pmatrix},
\]

where \( t \) is measured in the rest system of the \( K \) meson, and the state \( | \bar{K}^0 \rangle \) is defined to be

\[
| \bar{K}^0 \rangle = CP | K^0 \rangle.
\]

The \( CP \) operator can only be determined by the strong and the electromagnetic interactions up to a phase factor

\[
e^{i \chi},
\]

where \( S \) is the strangeness operator and \( \chi \) an arbitrary real number. However, since \( H_{\theta} \) is, by definition, the usual \( CP \)-conserving weak interaction, and since it must allow the \( \Delta S = \pm 1 \) transitions, this phase factor

\[14\text{ Cf. R. Sachs, Ann. Phys. (N. Y.) 22, 239 (1963). Sachs' notation is related to ours by } t = e^{i \phi}, s = cot(\phi/2). \text{ He uses a different normalization condition for the eigenstates, however.}\]
The states $\alpha, K^0$ (or $\bar{K}^0$) are eigenstates of the strong and electromagnetic interactions only, and $m_{\alpha}, m_{K^0}$ are the corresponding unperturbed eigenvalues.

We may express $(\Gamma+iM)$ in terms of the usual Pauli matrices $\sigma_1, \sigma_2, \sigma_3$:

\begin{equation}
(\Gamma+iM)=D+i(E_1\sigma_1+E_2\sigma_2+E_3\sigma_3),
\end{equation}

where

\begin{align*}
\sigma_1 &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \\
\sigma_2 &= \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \\
\sigma_3 &= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}
\end{align*}

and $D, E_1, E_2,$ and $E_3$ are four complex numbers. It is convenient to introduce the complex variables $E, \theta,$ and $\phi$:

\begin{align*}
E &= (E_1^2+E_2^2+E_3^2)^{1/2}, \\
E_1 &= E \cos \theta, \\
E_2 &= E \sin \theta \sin \phi, \\
E_3 &= E \sin \theta \cos \phi.
\end{align*}

The eigenstates $K^0$ and $\bar{K}^0$, defined by

\begin{equation}
(\Gamma+iM)|K^0\rangle = [\gamma_{j} + im_{j}]|K^0\rangle,
\end{equation}

where $j=1$ or 2 can be readily obtained; these solutions are identical in form to that of a spin-1/2 magnet in a magnetic field, and are given by

\begin{align*}
|K^0\rangle &= N_1 \begin{pmatrix} \cos \frac{\theta}{2} \\ e^{i\phi} \sin \frac{\theta}{2} \end{pmatrix}, \\
|\bar{K}^0\rangle &= N_1 \begin{pmatrix} \sin \frac{\theta}{2} \\ -e^{i\phi} \cos \frac{\theta}{2} \end{pmatrix},
\end{align*}

where

\begin{align*}
N_1 &= \left| \cos \frac{\theta}{2} \right| + \left| e^{i\phi} \sin \frac{\theta}{2} \right| \right|^{-1/2}, \\
N_2 &= \left| \sin \frac{\theta}{2} \right| + \left| e^{i\phi} \cos \frac{\theta}{2} \right| \right|^{-1/2}.
\end{align*}

The observed mass $m_j$ and width $\gamma_j$ of $K^0$ are related to $D$ and $E$ by

\begin{align*}
D &= \frac{1}{2}(\gamma_1 + \gamma_2) + \frac{i}{2}(m_1 + m_2), \\
iE &= \frac{1}{2}(\gamma_1 - \gamma_2) + \frac{i}{2}(m_1 - m_2).
\end{align*}

A useful parameter is

\begin{align*}
\alpha &= \langle K^0 | \bar{K}^0 \rangle \\
&= N_1 N_2 \left[ \cos \frac{\theta}{2} \sin \frac{\theta}{2} \right] - \left[ e^{i\phi} \sin \frac{\theta}{2} \right] e^{i\phi} \cos \theta \right|^{-1/2}.
\end{align*}

From the property that $\Gamma$ is a positive-definite Hermitian matrix, it follows that (independent of the validity of the CPT theorem)

\begin{equation}
|\alpha|^2 \leq \gamma_1 \gamma_2 \left[ (m_1 - m_2)^2 + \left( \gamma_1 + \gamma_2 \right)^2 \right],
\end{equation}

which is $\sim 1.3 \times 10^{-4}$, if $m_1 - m_2$ is set to be about equal to $\gamma_1$. 

\[ \text{(28)} \]
The states $K^0$ and $\bar{K}^0$ can be expressed as linear combinations of the eigenstates $K_1^0$ and $K_2^0$:

$$|K^0\rangle = N_1^{-1}\cos\theta |K_1^0\rangle + N_2^{-1}\sin\theta |K_2^0\rangle$$  \hspace{1cm} (42)

and

$$e^{i\phi} |\bar{K}^0\rangle = N_1^{-1}\sin\theta |K_1^0\rangle - N_2^{-1}\cos\theta |K_2^0\rangle.$$  \hspace{1cm} (43)

Let us consider a neutral $K$-meson beam which, at $t=0$, is in a pure $K^0$ state (or a pure $\bar{K}^0$ state). The fractional number of all $K$ mesons decaying per unit time at time $t$ is given by the familiar expression

$$\Psi_k(t) = A_{\pm} \gamma \exp(-\gamma t) + B_{\pm} \gamma \exp(-\gamma \bar{t})$$

$$+ (C_{\pm} [\gamma (1+\gamma)/\gamma - i(m_1 - m_2)])$$

$$\times \exp[-(1/\gamma + i\gamma)/(1+i\gamma)] + c.c.$$.  \hspace{1cm} (44)

The plus (or minus) sign in the subscripts denotes the case that the neutral $K$ meson is, at $t=0$, in a $K^0$ state (or a $\bar{K}^0$ state). The coefficients $A_+$, $A_-$, $B_+$, $B_-$ are real and $C_+$, $C_-$ are complex. These eight real coefficients depend only on two complex numbers $\theta$ and $\phi$:

$$A_+ = N_1^{-1}\cos\theta \gamma^2,$$

$$B_+ = N_2^{-1}\sin\theta \gamma^2,$$

$$A_- = N_1^{-1}(1-e^{-i\phi}) \sin\theta \gamma^2,$$

$$B_- = N_2^{-1}(1-e^{-i\phi}) \cos\theta \gamma^2,$$

$$C_+ = (N_1 \gamma)^{-1} (\cos\theta \gamma^2 \sin^2 \theta \alpha),$$

$$C_- = - (N_2 \gamma)^{-1} (\sin\theta \gamma^2 \cos \theta \gamma^2 \sin \theta \alpha).$$  \hspace{1cm} (45)

The integrated value of $N_\pm(t)$ over all $t$ must be 1; i.e.,

$$A_+ + B_+ + C_+ + C_- = 1.$$  \hspace{1cm} (46)

The constants $A_\pm$, $B_\pm$, and $C_\pm$ can be directly measured by studying the time development of such a $K$-meson beam. These measurements can be used to test the various symmetry or symmetry-violation properties of the $K_1^0$, $K_2^0$ system.

Remarks

(1) In general, the $(2 \times 2)$ matrix $(\Gamma + i M)$ depends on eight real parameters, of which four parameters can be determined by the observed widths $\gamma_1$, $\gamma_2$ and masses $m_1$, $m_2$ of $K_1^0$ and $K_2^0$. The remaining four parameters can be chosen to be the real and the imaginary parts of $\theta$ and $\phi$:

$$\theta = \theta_0 + i \theta_1;$$  \hspace{1cm} (47)

and

$$\phi = \phi_0 + i \phi_1.$$  \hspace{1cm} (48)

Three of these, $\theta_0$, $\theta_1$, and $\phi_1$, can be determined by measuring $A_\pm$, $B_\pm$, $C_\pm$ and by using Eq. (45). The real part of $\phi$ depends on the relative phase between $K^0$ and $\bar{K}^0$, which, as remarked earlier, is determined only if we use the usual $CP$-conserving weak interaction $H_0$ and the definition of $K_\pm$ given by Eq. (15).\footnote{For the simple phenomenological model, the effects of $H_0$ and $H_0$ are clearly separated and $\phi_0$ can be determined from experiments, as discussed in Sec. III and IV. For the other classes, this is less simple. Sometimes, it may be more convenient to define $\phi_0$ in a way that $\gamma_0$ and $\gamma_1$ are the same for the simple phenomenological model.}

(2) If $CPT$ invariance holds, then $\gamma_1 = \gamma_2$ and $M_{1\pm} = M_{2\pm}$. By using Eq. (28), it follows that $E_2 = 0$. Thus,

$$\theta = \frac{\pi}{4},$$

and

$$\frac{N_1}{N_2} = [\frac{1}{2} (1 + |e^{i\phi}|^2)] \approx \frac{1}{2}$$

and $\alpha$ is real. The validity of the $CPT$ theorem can be tested by measuring the constants $A_\pm$, $B_\pm$, $C_\pm$ in Eq. (44) and observing that

$$A_+ = B_+ = A_- e^{i\phi} = B_- e^{-i\phi}.$$

and

$$C_- = - C_+ e^{i\phi}$$

If $CPT$ invariance holds, the eigenstates $K_1^0$ and $K_2^0$ can also be expressed in the forms given by Eqs. (9) and (10) which depend only on one complex parameter

$$e^{i\phi} = (1 - e^{i\phi})/(1 + e^{i\phi}).$$  \hspace{1cm} (56)

(3) Independent of $CPT$ invariance, if $T$ invariance holds, then

$$\Gamma_{12}^* / \Gamma_{12} = M_{12}^* / M_{12}.$$  \hspace{1cm} (57)

Thus, the imaginary part of $\phi$ must be zero; i.e.,

$$\phi_1 = 0.$$  \hspace{1cm} (58)

By using Eqs. (36), (37), and (45), we find

$$N_1 = N_2,$$

$\alpha$ is pure imaginary,

$$A_\pm = B_\mp,$$

and

$$C_+ = C_-*.$$  \hspace{1cm} (60)

Equations (59) and (60) can be used as tests of $T$ invariance (without any assumption concerning the invariance of $CPT$, or $CP$, or $C$, or $P$).

(4) Independent of $CPT$ invariance, if $F/G$ is $\sim 10^{-8}$, then by using Eqs. (20)–(25) and Eq. (27), it is easy to see that the accuracy of keeping $F/G$, but neglecting $F/G$, all $CP$ violations and $CPT$ violations are due to the mass operator $M$. As already discussed in the previous section, to this assumed accuracy, such a theory is identical with the simple phenomenological model in which, except the mass operator $M$ of the $K_1^0$, $K_2^0$ system, all other transition matrix elements satisfy the consequences of $CP$ invariance and $T$ invariance. Thus

$$\Gamma_{12} = \Gamma_{12}^*$$

is real and $\Gamma_{11} = \Gamma_{22}$, the relative phase between $K^0$ and $\bar{K}^0$ such that the amplitudes for $K^0$ and $\bar{K}^0$ going to the $I=0$ two-pion state are relatively real. [A method of determining $\phi_0$ by using this convention has been discussed by T. T. Wu and C. N. Yang, Phys. Rev. Letters 13, 380 (1964).] The two definitions of the relative phase between $K_1^0$ and $K_2^0$ are the same for the simple phenomenological model.
which correspond to $E_2$, real (61)
and $E_3$, real. (62)
The further consequences of this simple phenomenological model will be studied in the next two sections.

III. CONSEQUENCES OF THE SIMPLE PHENOMENOLOGICAL MODEL AND CPT INVARIANCE

In the case of the simple phenomenological model, if CPT invariance holds, the eigenstates $K^\circ$ and $K^e$ are given by Eqs. (9) and (10). By using Eqs. (31), (39), (52), and (56), the parameter $\epsilon$ is given by

$$2\epsilon / (1 - \epsilon^2) = (E_2 / |E|) e^{iq},$$

where the phase $\beta$ is defined by

$$iE = |E| e^{-iq} = \frac{1}{2}(\gamma_1 - \gamma_2) + \frac{1}{2}(m_1 - m_2).$$

According to Eq. (61), $E_2$ is real. Thus, the phase $\beta$ and the magnitude $|\epsilon|$ of the parameter $\epsilon$ satisfy

$$\sin \beta = |\epsilon|^2 \sin (2\beta + \delta),$$

$$\cos \beta = |\epsilon|^2 \cos (2\beta + \delta).$$

In this model, all the transition matrix elements for the decays of $K^\circ$ and $K^e$ into any continuum states, such as $2\pi$, $3\pi$, etc., satisfy the consequences of CPT invariance. The magnitude $|\epsilon|$ is, therefore, given by Eq. (1). Consequently, both $|\epsilon|$ and $\delta$ have been experimentally measured. If we neglect $|\epsilon|^2$ in Eq. (65), the phase $\delta$ is given by Eq. (12):

$$\tan \delta = -2(m_1 - m_2) / (\gamma_1 - \gamma_2),$$

which has always two solutions, $\delta = \beta$ and $\delta = \pi + \beta$. There are, at present, some discrepancies between the existing measurements$^{17}$ of the mass difference $(m_1 - m_2)$; therefore, the exact value of $\beta$, or $\delta$, is not yet definite.

The following is a list of some CPT-violating effects in this simple model under the assumption of CPT invariance$^{18}$:

17 The experimental value of the mass difference between $K^\circ$ and $K^e$ varies from 0.80 to 1.9 $\psi$. See B. Aubert et al., Phys. Letters 10, 215 (1964); U. Camerini et al., Phys. Rev. 128, 362 (1962); J. H. Christensen et al., Conference on Fundamental Aspects of Weak Interactions, BNL, New York (1963); J. H. Christensen, Tech. Rept. 34, Princeton University (1964); V. L. Fitch, Nuovo Cimento 22, 1160 (1961); M. L. Good, et al., Phys. Rev. 124, 1223 (1961). In Ref. 12, the lowest of these values was used to obtain a numerical value for the ratio in Eq. (70).

18 While the equalities given by Eqs. (66)–(72) can be used to test the validity of the simple model (i.e., $F = 10^{-6}$), it is important to notice that some of these results are also valid in other cases. For example, if we consider the case $F = 10^{-6}$ and assume that $H_1$ and $H_2$ both satisfy the $|\Delta I| = 1$ rule for the strangeness-nonconserving amplitudes. The $|\Delta I| = 1$ amplitude is then, due to the combined effect of electromagnetic interaction and either $H_1$ or $H_2$. Thus, neglecting $O(\alpha)$, the equality

$$\text{Rate}(K^\circ \to \pi^+ + \pi^-) / \text{Rate}(K^\circ \to 2\pi^0) = \text{Rate}(K^e \to \pi^+ + \pi^-) / \text{Rate}(K^e \to 2\pi^0)$$

remains valid. Assuming CPT invariance and the $\Delta Q = \Delta S$ rule

$$\text{Rate}(K^\circ \to \pi^+ + \pi^-) / \text{Rate}(K^\circ \to 2\pi^0) = |\epsilon|^2,$$

$$\text{Rate}(K^2 \to 3\pi^0) = |\epsilon|^4,$$

The decay of $K^\circ \to \pi^+ + \pi^- + \pi^0$ can occur with the final 3$\pi$ system in either a $CP = -1$ state (which shows CP violation) or a $CP = 1$ state (which is compatible with CP conservation). The amplitude of $K^\circ \to \pi^+ + \pi^- + \pi^0$ in a final $CP = -1$ state is $\epsilon$ times the corresponding amplitude of $K^e$ decay. The amplitude of $K^\circ \to \pi^+ + \pi^- + \pi^0$ in a final $CP = +1$ state does not violate CP conservation, but is reduced from the corresponding $CP = -1$ amplitude of $K^e$ decay only by the centrifugal barrier effects.

(3) For leptonic decays, we assume that the $\Delta Q = \Delta S$ rule holds for the transition matrix elements between the $K^\circ$, $K^e$ states and any leptonic states [i.e., the $\Delta Q = \Delta S$ rule holds for $H_0$].

$$\text{Rate}(K^\circ \to \pi^+ + l^- + \bar{\nu}_l) = \text{Rate}(K^\circ \to \pi^+ + l^- + \bar{\nu}_l),$$

$$\text{Rate}(K^e \to \pi^+ + l^- + \nu_l) = \text{Rate}(K^e \to \pi^- + l^+ + \nu_l),$$

and

$$\text{Rate}(K^\circ \to \pi^+ + l^- + \bar{\nu}_l) = \text{Rate}(K^e \to \pi^- + l^+ + \nu_l),$$

where $l = e$ or $\mu$. Identical relations also hold for $K_4$ decays, provided we replace $\pi^+$ and $\pi^-$ in the above formulas by $(\pi^+ + \pi^-)$ and $(\pi^+ + \pi^-)$, respectively.

(4) By using Eqs. (52) and (56), the coefficients $A_\pm$, $B_\pm$, and $C_\pm$ in Eq. (44) are given by

$$A_\pm = B_\pm = (1 + |\epsilon|^2) / 2 |1 \pm \epsilon|^2 \pm 1 / (1 \pm |\epsilon|^2) \cos \delta,$$

and

$$C_\pm = \pm (\epsilon + *e) / (1 \pm |\epsilon|^2) \pm 1 / (1 \pm |\epsilon|^2) \cos \delta.$$
tains only one real parameter which has not yet been measured.

It is convenient to represent the \( K^0 \) and \( \bar{K}^0 \) states by

\[
\langle K^0 \rangle = \left[ 2(1 + |\epsilon_1|^2) \right]^{-1/2} \left( \begin{array}{c} 1 + \epsilon_1 \\ 1 - \epsilon_1 \end{array} \right),
\]

(73)

and

\[
\langle \bar{K}^0 \rangle = \left[ 2(1 + |\epsilon_2|^2) \right]^{-1/2} \left( \begin{array}{c} 1 + \epsilon_2 \\ -(1 - \epsilon_2) \end{array} \right),
\]

(74)

where \( \epsilon_1 \) and \( \epsilon_2 \) are related to \( \theta \) and \( \phi \) by

\[
(1 - \epsilon_1)/(1 + \epsilon_1) = \cos \theta \tan \phi/2
\]

(75)

and

\[
(1 - \epsilon_2)/(1 + \epsilon_2) = \cos \phi \cot \theta,
\]

(76)

which can, in turn, be expressed in terms of the real parameters \( E_2 \) and \( E_2 \).

We note that

\[
\frac{\text{Rate}(K^0 \to \pi^+ + \pi^-)}{\text{Rate}(K^0 \to 2\pi^0)} = \frac{\text{Rate}(K^0 \to \pi^+ + \pi^-)}{\text{Rate}(K^0 \to 2\pi^0)}
\]

\[
= \frac{1 + |\epsilon_1|^2}{1 + |\epsilon_2|^2}
\]

(77)

and

\[
\frac{\text{Rate}(K^0 \to 3\pi^0)}{\text{Rate}(K^0 \to 3\pi^0)} = \frac{1 + |\epsilon_1|^2}{1 + |\epsilon_1|^2}
\]

(78)

From Eq. (77) and the experimental result\(^t\) of Christenson \textit{et al.}, it follows that

\[
|\epsilon_2|^2 \leq |\epsilon|^2 \approx 4.8 \times 10^{-6},
\]

or

\[
|\epsilon_1| \leq |\epsilon| \times \sqrt{1 - |\epsilon|^2} \approx 2.2 \times 10^{-6},
\]

(79)

which, together with the inequality (41), implies that

\[
|\epsilon_1| \leq \sqrt{3.6 \times 10^{-5}}.
\]

(80)

Thus, \( |\epsilon_1| \) and \( |\epsilon_2| \) must both be small numbers, and \( |\epsilon_1|^2 \), \( |\epsilon_2|^2 \) and \( |\epsilon_1| \times |\epsilon_2| \) may be neglected as compared to 1. In this approximation, all formulas become considerably simpler. The results are given below:

(1) Let

\[
\epsilon_2 = |\epsilon_2| \exp(i\delta); \]

(81)

then

\[
|\epsilon_1| = |\epsilon_2| = |\epsilon|
\]

(82)

and

\[
\frac{1}{2}(\delta_1 + \delta_2) = \delta,
\]

(83)

where

\[
\delta = \beta \quad \text{or} \quad (\pi + \beta).
\]

(84)

The constants \( |\epsilon| \) and \( \beta \) have both been measured, and are given, respectively, by Eqs. (1) and (64).

The entire problem contains only one unknown real parameter which may be chosen to be

\[
\frac{1}{2}(\delta_1 - \delta_2) = \eta.
\]

(85)

(2) The coefficients \( A_\pm \), \( B_\pm \), and \( C_\pm \) in Eq. (44) are given by

\[
A_\pm = \frac{1}{2} \left[ 1 \mp 2 |\epsilon| \cos(\delta - \eta) \right],
\]

(86)

\[
B_\pm = \frac{1}{2} \left[ 1 \mp 2 |\epsilon| \cos(\delta + \eta) \right],
\]

(87)

and

\[
C_\pm = \pm |\epsilon| \cos \eta e^{-i\eta}.
\]

(88)

(3) For the \( K^0 \) decay into pions, Eqs. (66) and (67) remain valid. For the \( K^0 \) decay into leptons, if \( \Delta Q = \Delta S \) rule holds, we have

\[
\text{Rate}(K^0 \to \pi^- + l^+ + \nu_l)/\text{Rate}(K^0 \to \pi^+ + l^- + \bar{\nu}_l) = 1 + 4 |\epsilon| \cos \delta_l,
\]

(89)

where \( l = e \) or \( \mu \) and \( j = 1 \) or 2. The same formula holds for \( K^0 \) decays, provided \( \pi^\pm \) is replaced by \( (\pi^+ + \pi^-) \).

(4) If \( T \) invariance holds (but not \( CPT \) invariance), then by using Eqs. (58), (75), and (76), and since \( \cos \theta = 0 \), we find

\[
\eta = \frac{1}{2} \pi.
\]

(90)

(5) It has been suggested\(^d\) that the observation of \( K^0 \to \pi^+ + \pi^- \) can be compatible with \( CP \) invariance, provided there exists an energy difference \( \Delta E \) between the \( K^0 \) and \( \bar{K}^0 \) states due to some cosmological background. The apparent \( CP \) noninvariance is attributed not to the basic violation of the interaction, but is regarded as the result of the initial (or boundary) condition of our universe which gives rise to this background field. As remarked in the paper by Bernstein \textit{et al.},\(^3\) the amount of this energy difference \( \Delta E \) is proportional to \( E_2^j \), where \( J = \text{spin of the cosmological background field} \) (or, rank of the corresponding field tensor) and \( E \) is the energy of the \( K \) meson measured in the rest system of the cosmological background field.

From a phenomenological point of view, except for this energy dependence of \( \Delta E \), the description of the \( K^0 \) and \( \bar{K}^0 \) system in such a cosmological background field is the same as the simple model (without \( CPT \) invariance) considered in this paper, provided

\[
e_2 = - e_1;
\]

(91)

\[i.e., \eta = \frac{1}{2} \pi,
\]

which is identical with Eq. (90). If \( J = 0 \), then there is no difference between the cosmological possibility with \( CP \) invariance and the case of an extremely weak \( CP \)-noninvariant, \( CPT \)-noninvariant, but \( T \)-invariant \( H \)\(^7\) with \( F \approx 10^{-5} \), unless either the measurement of the appropriate transition amplitudes can be improved to include terms of the order of \( 10^{-6} \), or the observation of a real emission of the cosmological field quantum can become feasible.

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Possible $C, T$ Noninvariance in the Electromagnetic Interaction

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The experimental foundation of the various discrete symmetry properties for the strong and the electromagnetic interactions is reviewed. It is found that there is strong evidence that both interactions are invariant under $P$ and $CPT$, and good evidence that the strong interaction is invariant under $C$ and $T$. However, at present, there is a complete lack of evidence that the electromagnetic interaction of the strongly interacting particles is invariant under $C$ and $T$. Possible experiments that can test such $C, T$ invariance or noninvariance are discussed. It is pointed out that if the electromagnetic interaction of the strongly interacting particles has strong violations of $C, T$ invariance, then, through second-order processes, $C$-violating and $CPT$-violating effects can be observed in the usual strong and weak processes, respectively. In particular, the decay $K^+_s \rightarrow \pi^+ + \pi^-$ may occur with an amplitude $\sim (\alpha/\pi)$ times that of $K^+_s \rightarrow \pi^+ + \pi^-$.  

I. INTRODUCTION

SINCE the discovery\footnote{This research was supported in part by the U. S. Atomic Energy Commission and the National Science Foundation.} that

\begin{equation}
K^+_s \rightarrow \pi^+ + \pi^-,
\end{equation}

in apparent violation of $CP$ invariance, it has become clear that the experimental foundation of all discrete symmetries should be re-examined for all the interactions. Such a study will be made in this paper for both the strong and electromagnetic interactions. We shall see that there is strong evidence supporting the premise that both interactions preserve $P$ and $CPT$ to a great accuracy; furthermore, the strong interaction is invariant under $T$ to a fair degree of accuracy ($\sim 2\%$), and through $CPT$ and $P$ invariance the same accuracy applies also to $C$ conservation in the strong interaction. The situation is, however, very different for the electromagnetic interactions. As will become clear, at present there exists no evidence that the electromagnetic interactions of the strongly interacting particles are invariant under $C$ and $T$. Indeed, all existing experimental results are compatible with the possibility of a very large violation of $C$ and $T$ in the electromagnetic interaction of these strongly interacting particles. Throughout this paper, the three operators $C$, $P$, and $T$ denote, respectively, the usual particle-antiparticle conjugation, space inversion, and time reversal.  

For definiteness, let us write the nonleptonic part of the electromagnetic current operator $j_\mu$ as

\begin{equation}
j_\mu = J_\mu + K_\mu,
\end{equation}

where

\begin{equation}
CJ_\mu C^{-1} = -J_\mu
\end{equation}

and

\begin{equation}
CK_\mu C^{-1} = +K_\mu.
\end{equation}

Both $J_\mu$ and $K_\mu$ are assumed to have the same transformation properties under $CPT$, and both are vector currents under $P$ (in the absence of the weak interaction). The condition of $C$, or $T$ invariance is

\begin{equation}
K_\mu = 0.
\end{equation}

We note that if $K_\mu \neq 0$ and if some of its matrix elements are comparable in magnitude to those of $J_\mu$ (i.e., large $C$, $T$ violation), then, through virtual electromagnetic processes, all strong reactions may violate $C$ and $T$ to order of $\alpha$ where $\alpha = (137)^{-1}$.  

It has been suggested recently\footnote{T. D. Lee and L. Wolfenstein, Phys. Rev. 138, B1400 (1965).},\footnote{L. B. Okun (unpublished report). See also J. Frenkel and M. Veliman, Phys. Letters 15, 88 (1965), in which they consider the possibility that the $C, T$-noninvariant interaction is simply the usual $SU_3$-violating, but $SU_3$-conserving part of the strong interaction. This possibility seems to encounter several difficulties, especially in view of the present accuracy ($\sim 2\%$ in relative amplitude) of $T$ invariance in many nuclear reactions (see Ref. 12). In all such reactions, large violations of $SU_3$ symmetry may occur because of the difference between the Compton wavelengths of $\pi$ and $K$. It is also difficult to see how such a strong violation of $C, T$ invariance manages to produce only a very small $CP$-noninvariant amplitude in the decay $K^+_s \rightarrow \pi^+ + \pi^-$.} in connection with reaction (1), that the violation of $CP$ invariance is not due to the usual weak interaction, but, rather, is due to the possible existence of a new $C, T$ noninvariant interaction called $H_F$, whose coupling constant $F$ is much stronger than the Fermi constant $G_F$ of the usual weak interaction $H_W$. It is estimated that

\begin{equation}
P \sim 10^3 G_F;
\end{equation}

or, the dimensionless constant ($Fm_\pi$) is given by

\begin{equation}
Fm_\pi \sim 10^{-2},
\end{equation}

where $m_\pi$ is the mass of the proton. The usual weak interaction violates $C$ invariance and $P$ invariance, but it is assumed to be invariant under $CP$ and $T$. The new interaction $H_F$ is assumed to violate $C$ invariance and $T$ invariance, but it conserves strangeness, and is invariant under $CT$ and $P$. The decay $K^+_s \rightarrow 2\pi$ can occur through
H_{\phi\phi}\) alone, but reaction (1) can occur only through the second-order CP-noninvariant term \(H_{\phi}\), thus, its amplitude is much smaller than that of \(K_{\phi}^2 \rightarrow 2\phi\).

If such an \(H_{\phi}\) exists, then, at least to order \((Fm_{\phi}^2)\), an “effective” \(C, T\)-noninvariant current \(K_{\phi}\) will appear. However, total lack of evidence for \(C, T\) invariance for the nonleptonic part of the electromagnetic interaction suggests another more interesting possibility. The electromagnetic interaction may itself show a large violation of \(C\) and \(T\), and the “new” interaction \(H_{\phi}\) may well be simply a manifestation of the second-order electromagnetic effects. This explains why the magnitude of \((Fm_{\phi}^2)\) is \(\sim \alpha\).

While these are merely some hypothetical possibilities, they do reflect our present state of ignorance and should, therefore, provide an incentive for a critical study of the various experimental implications of possible \(C, T\) violations in the electromagnetic interactions of strongly interacting particles. To set a sensitive limit on the magnitude of possible \(C\) noninvariances, experiments should be done to measure the rates of

\[
\psi^0 \rightarrow \pi^0 + e^+ + e^-
\]

or

\[
\phi^0 \rightarrow \phi^0 (\text{or } \phi^0) + \gamma,
\]

and to study the possible \(\pi^0\) asymmetry in

\[
\omega^0 (\text{or } \eta^0) \rightarrow \pi^+ + \pi^- + \gamma,
\]

etc. For a direct limit on possible \(T\) noninvariance in the electromagnetic interactions, a reaction such as

\[

\]

is important. A systematic phenomenological discussion of these and similar reactions will be given in the subsequent sections.

The electromagnetic interaction is usually thought to be invariant under \(C, P, \) and \(T\) separately. There exists a so-called “minimal” principle, which states that if, in the absence of the electromagnetic field \(A_\mu\), the Lagrangian \(\mathcal{L}\) is known, then the replacement of \((\partial / \partial \phi_n)\) by \((\partial / \partial \phi_n - ieA_\phi)\) changes \(\mathcal{L}\) to another Lagrangian \(\mathcal{L}_\phi\), which contains all the dependence on the electromagnetic field. A consequence of this “minimal” principle is that if \(\mathcal{L}\) is invariant under \(T\), so must \(\mathcal{L}_\phi\) be, since \(\partial / \partial \phi_n\) and \((\partial / \partial \phi_n - ieA_\phi)\) transform in the same way under \(T\).

It must be pointed out that the validity of such a “minimal” principle for the strongly interacting particle is, as yet, unclear, and it will not be assumed in this paper. Throughout our discussions, we assume that the electromagnetic interactions of the leptons are invariant under \(C, P, \) and \(T\) separately, but the electromagnetic interactions of the nonleptons can have strong violations of \(C, T\) invariance.

II. PRESENT EXPERIMENTAL LIMITS ON C, P, T INVARIANCES FOR STRONG INTERACTIONS AND ELECTROMAGNETIC INTERACTIONS

Since the discovery of CP violation in weak interactions, there have been many experiments to search for possible P-violating effects in nuclear reactions. These experiments establish that the magnitudes of the \(P\)-nonconserving amplitudes are smaller than that of the corresponding \(P\)-conserving amplitudes by, at least, a factor \(\sim 10^{-4}\), which is about the order of magnitude of the dimensionless weak coupling constant \(g_{\gamma}\). Thus, we should regard both strong and electromagnetic interactions to be \(P\) conserving (neglecting the weak-coupling constant).

Invariance under \(CPT\) implies mass and lifetime equalities between any particle and its antiparticle. In view of the possibly large violations of \(C\) and \(CP\) invariances, we may use such equalities as evidence for \(CPT\) invariance. Among such equalities, the most accurate one is that between \(K^0\) and \(\bar{K}^0:\)

\[
\langle K^0 | H_{\phi\phi} + H_{\phi\phi} | K^0 \rangle = \langle \bar{K}^0 | H_{\phi\phi} + H_{\phi\phi} | K^0 \rangle,
\]

where \(H_{\phi\phi}\) and \(H_{\phi\phi}\) are, respectively, the Hamiltonians for the strong and the electromagnetic interactions. From the experimental mass difference \(\Delta m\) between \(K^0\) and \(\bar{K}^0\), we conclude that Eq. (8) holds to \(\sim |\Delta m/2\pi| \sim 10^{-14}\). In the following, we take \(H_{\phi\phi}\) and \(H_{\phi\phi}\) to be invariant under \(P\) and \(CPT\); therefore, to the same great accuracy, both interactions are also invariant under \(C\).

The present experiments on the polarization and accurately described by the usual form \(i\xi^0 \gamma_\mu A_\mu A_\phi\), which is invariant under \(C, P, \) and \(T\) separately.


angular asymmetry in the $p-p$ scattering gives an upper limit on the magnitude of the $T$-noninvariant amplitude of not more than a few percent of the $T$-invariant amplitude. A corresponding upper limit of $\approx 2\%$ has also been obtained from the existing experiments on reciprocity relations$^{9}$ in low-energy nuclear reactions. Thus, the strong interaction has been found to be invariant under the time-reversal operation to within a few percent. By using $CT$ invariance, we can conclude that the strong interaction is also invariant under $C$ to within a few percent.

The question of $C, T$ invariance in the electromagnetic interaction for the strongly interacting particles is, however, a completely open one. We note that the electromagnetic interaction of a single physical nucleon is characterized by the matrix element $\langle N'|g_{\mu}(x)|N\rangle$, which, by invariance under the continuous Lorentz transformations and space reflection, must be of the form

$$\langle N'|g_{\mu}(x)|N\rangle = i\epsilon U_{\mu'N'}\gamma_{\mu}F_{\mu} + (N'_\mu + N_\mu)F_{\mu} + (N'_\mu - N_\mu)F_{\mu} \times \exp[-i(N'_\mu - N_\mu)x_\mu],$$

where $N_\mu$ and $N'_\mu$ are, respectively, the 4-momenta of the initial and the final single physical nucleon states $\langle N'|$ and $|N\rangle$, $U_N$ and $U_{N'}$ are the solutions of the free Dirac equations with $N_\mu$ and $N'_\mu$ as their respective 4-momenta. The $\gamma_\mu, \gamma_\tau, \gamma_\rho, \gamma_\sigma$ are the usual Dirac matrices and $F_\tau, F_\rho, F_\sigma$ are functions of the square of the 4-momentum transfer

$$q^2 = (N'_\mu - N_\mu)^2$$

only. [We use for a 4-vector $N_\mu = (N, iN_\mu)$, where $N_\mu$ is real.] From Hermiticity, we find $F_\tau, F_\rho, F_\sigma$ to be all real. If $T$ invariance holds, then $F_\tau$ and $F_\rho$ are real, but $F_\sigma$ is pure imaginary; thus,

$$F_\sigma = 0$$

implies $T$ noninvariance. However, because of the conservation of current, the $T$-noninvariant term $F_\sigma$ actually vanishes on the nucleon mass shells.$^{10}$ The electromagnetic form factors of the nucleon can be measured by studying the scattering of

$$l^\pm + N \to l^\pm + N,$$

where $l = e$ or $\mu$, in which the nucleons are on their mass shells. Thus, neglecting higher order electromagnetic effects, no information concerning the possible $C, T$ noninvariance in the operator $g_{\mu}$ can be obtained from reaction (12).

The electromagnetic property of a nucleus is determined by the corresponding properties of its constituents. It is a good approximation (i) to regard the nucleus as a collection of physical nucleons interacting through their strong nuclear forces, and (ii) to neglect other terms which can occur in the nucleon-photon vertex when the nucleons are off the mass shell. Within these approximations, and neglecting higher order radiative processes, there are still no $T$-noninvariant terms which contribute to any nuclear $\gamma$ transition. The accuracy of these two approximations may be crudely estimated to be $\sim O(\alpha^2/m_{\gamma}^2)$ for (i), at least for $T$-violating effects, and $\sim O(V/m_{\gamma})$ for (ii), where $V$ is the nuclear potential energy, $m_{\gamma}$, $m_e$ are, respectively, the masses of the nucleon and the pion. Thus, to the accuracy of $\sim 0.1$ percent, no information concerning $C, T$ invariance of $H_{\gamma}$ can be obtained by studying the nuclear photoprocesses; rather, such processes can only be used to establish more firmly the $C, T$ invariance of the strong interaction.

If $H_{\gamma}$ strongly violates $C, T$ invariance, then, through higher order effects, the reciprocity relation for any strong reaction may be violated with a fractional difference of the order of $10^{-4}$. All nuclear matrix elements in either $\gamma$ or $\beta$ transitions may also contain a small admixture of $T$-noninvariant amplitude which is $\sim (10^{-4} \times 10^{-4})$ times the $T$-invariant amplitude. Such a $T$-noninvariant amplitude can be observed by measuring the relative phase$^{14}$ between $G_F$ and $G_A$ in $\beta$ decay, or by studying, e.g., the $E2, M1$ interference term through the detection of a $[k_{\nu}(k_{\nu} \times k_{\beta})](k_{\nu} \cdot k_{\beta})$ term in an appropriate $\beta - \gamma$ transition.$^{14}$

Reciprocity relations such as $\gamma + A \leftrightarrow B + C$ can be used to test possible $T$ noninvariance in $H_{\gamma}$, where $A, B, C$ are any strongly interacting particle states. However, because of Hermiticity, a detailed spin-momentum analysis is necessary to test the reciprocity relation. As possible examples for such reciprocity tests, we may list $\gamma + n \leftrightarrow p + \pi^0$ or $\gamma + p \leftrightarrow n + \pi^+$. Other direct tests of the $C, T$ invariance properties of $H_{\gamma}$ can be obtained by studying the relevant decays of mesons and hyperons. These will be analyzed in detail in the subsequent sections.

### III. DECAYS OF THE PSEUDOScalar MESONS

Since $H_{\mu}$ does conserve $C$, its eigenstates such as $\pi^0$ and $\rho$ are also eigenstates of $C$. In view of the possibility that $H_{\mu}$ may not be invariant under $C$, we cannot use the $\gamma$ decay modes to determine the $C$ values of these mesons. For definiteness, we assume that virtual

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9 We would like to thank Professor R. Serber and Professor G. C. Wick for pointing this out, and for enlightening discussions.
10 M. T. Burgoy et al., Phys. Rev. Letters 1, 324 (1958). It should be emphasized that, independent of $T$ invariance, $(G_F/G_V)$ must be real if the nonleptonic current $(V_{e} + A_{e})$ in the weak interaction satisfies the charge symmetry condition; i.e., $[V_{e} + A_{e}] = -[\exp(i\theta_{e})] [V_{e} + A_{e}] [\exp(-i\theta_{e})]$, where $I_{Y}$ is the $Y$ component of the isospin operator, $\theta$ denotes the Hermitian conjugation if $\mu = \frac{1}{2}$ and $(-1)$ times the Hermitian conjugation if $\mu = \frac{1}{2}$. For a detailed discussion, see E. H. Benley and B. A. Jacobsen, Phys. Rev. Letters 113, 225 (1959); B. A. Jacobson and E. H. Benley, Phys. Rev. Letters 113, 225 (1959).
transitions such as
\[ N + \bar{N} \rightarrow \pi^0 \]
(13)
can occur through \( H_\pi \); thus,
\[ C_{\pi^0} = +1. \]
(14)

A further confirmation of Eq. (14) is given by Eq. (46) below. (The validity of many of our subsequent conclusions on tests of \( C, T \) invariance depends, however, only on the relative \( C \) values of different mesons states.)

It is convenient to decompose the \( C = \mp 1 \) currents \( J^\mu \) and \( K^\mu \) into various components which have different transformation properties under the isotopic spin rotations:
\[ J^\mu = J^\mu_s + J^\mu_t \]
(15)
and
\[ K^\mu = K^\mu_s + K^\mu_t + \cdots, \]
(16)
where the superscripts \( s \) and \( t \) indicate that these currents transform as isoscalar \((I = 0)\) or isovector \((I = 1)\), respectively.\(^{16}\) If
\[ K_{\mu} \neq 0, \]
(17)
then \( H_\pi \) is not invariant under either \( C \) or \( T \). A conclusive test of \( C, T \) noninvariance in \( H_\pi \) would be the discovery of any process whose existence implies Eq. (17).

1. \( \pi^0 \) Decay
To first order in \( g_\rho \), the decay \( \pi^0 \rightarrow e^+e^- \) is forbidden because of current conservation. To second order in \( g_\rho \), the decays
\[ \pi^0 \rightarrow 2 \gamma, \]
(18)
\[ \pi^0 \rightarrow \gamma + e^+e^-, \]
(19)
and
\[ \pi^0 \rightarrow 2 e^+e^-, \]
(20)
contain only matrix elements of \( J^\mu_s, J^\mu_t \), and \( K^\mu_s, K^\mu_t \). Therefore, these reactions do not give direct tests of possible \( C, T \) noninvariance.
The decay
\[ \pi^0 \rightarrow 3\gamma \]
(21)
can occur only if \( C \) invariance is violated. Assuming that \( K_\mu \neq 0 \), its rate can be estimated. We find
\[
\frac{\text{Rate}(\pi^0 \rightarrow 3\gamma)}{\text{Rate}(\pi^0 \rightarrow 2\gamma)} \sim \frac{\alpha}{(\text{phase space})_\pi}(kR)^6 \sim 3 \times 10^{-6}, \]
where \((e^2/4\pi) = \alpha, k \) is the average momentum of the \( \gamma \) rays, the ratio of phase space is \( \sim (4\pi)^{-1}((m_\pi R)^2) \), and
\[ R \text{ is taken to be } \sim m_\pi^{-1}. \] The present experimental limit\(^{10}\) is only
\[ \frac{\text{Rate}(\pi^0 \rightarrow 3\gamma)}{\text{Rate}(\pi^0 \rightarrow 2\gamma)} \text{exp} < 3.8 \times 10^{-4}. \]
(23)
Thus, pion decays seem to be particularly insensitive to any possible violation of \( C, T \) in \( H_\pi \).

2. \( \eta^0 \) Decays
From the decays
\[ \rho^0 \rightarrow \pi^+\pi^- \]
(24)
and
\[ \rho^0 \rightarrow \eta^0 \pi^0, \]
(25)
we may conclude
\[ C_\eta = -1, \]
(26)
and
\[ C_\pi = -C_\eta C_{\pi^0}. \]
(27)
Thus, it follows that
\[ C_\pi = C_{\eta^0}. \]
(28)
The same conclusion would follow if we assume virtual transitions such as \( N + \bar{N} \rightarrow \eta^0 \) to be allowed.

The \( \eta^0 \) can only decay through \( H_\pi \). The decay
\[ \eta^0 \rightarrow e^+e^- \]
(29)
through a single-photon intermediate state violates \( C \) invariance, and it can occur only if the isovector part \( K_{\eta^0} \neq 0 \). The general matrix element of \( g_\rho \) for this transition is given by
\[ \langle \pi^0 | g_\rho(x) | \eta^0 \rangle = \langle \pi^0 | K^\mu_s(x) | \eta^0 \rangle = \int f_1(n_\eta x + n_\pi x) f_2(n_\eta x - n_\pi x) \]
\[ \times [4m_\eta m_\pi]^2 \exp[i(x_\eta - x_\pi)^2], \]
where \( n_\eta \) and \( n_\pi \) are, respectively, the 4-momenta of the initial \( \eta^0 \) and the final \( \pi^0 \) states, \( m_\eta \) is the mass of \( \eta^0, \omega_\pi \) is the energy of the pion, and \( f_1 \) and \( f_2 \) are form factors depending only on the square of the 4-momentum transfer,
\[ q^2 = (n_\eta - n_\pi)^2. \]
(30)
Conservation of current requires that
\[ f_1(m_\eta^2 - m_\pi^2) = q^2 f_2, \]
(32)
where \( m_\pi \) is the mass of \( \pi^0 \). To obtain an estimate of the decay rate, we assume\(^{17}\)
\[ f_1 = -\frac{1}{8} e^2(\pi^0)^2, \]
(33)
\[ ^{16} \text{Note added in proof. From a phenomenological point of view, one should also decompose } K_\eta = (K_{\eta^0}) + (R_{\eta^0}) + \cdots, \text{ where } (K_{\eta^0}) \text{ and } (R_{\eta^0}) \text{ transform, respectively, like a unitary singlet and a member of the } SU(2) \text{ group under the } SU(2) \text{ group of transformations. If } K_{\eta^0} = (K_{\eta^0}) + (R_{\eta^0}) \text{ only, then, in the limit of perfect } SU(2) \text{ symmetry, } \pi^0 \rightarrow e^+e^- \text{ and there is no } T \text{-invariant effect in } \gamma^\mu - g^\mu, e^+e^- + e^-e^- \text{. If } K_\eta = (K_{\eta^0}), \text{ then there are many additional forbidden processes such as } \psi^0 \rightarrow \omega_\pi + \gamma, \psi^0 \rightarrow \rho^+ + \gamma, \psi^0 \rightarrow \rho^0 + \gamma \text{ and the absence of } \pi^0, \pi^- \text{ asymmetries in } \psi^0 \rightarrow \pi^+ + \pi^- \text{ or } \gamma. \]
\[ ^{17} \text{D. Cline and R. M. Dowd, Phys. Rev. Letters 14, 530 (1965).} \]
\[ ^{18} \text{Such a form factor, after eliminating the photon propagator, gives an "effective" } C, T \text{-invariant point interaction: } \]
\[ \langle \pi^0 | i \gamma_\mu \gamma_5 \phi_\pi(\partial \phi_{\eta^0}) - \phi_\pi(\partial \phi_{\eta^0}) \phi_\pi \rangle, \]
where \( \phi_\pi, \phi_{\eta^0}, \psi_\pi, \psi_{\eta^0} \) are the operators for \( \pi^0, \eta^0, \pi^0, \eta^0 \). In this paper, we assume that there does not exist any additional direct strangeness conserving interaction between } e^+, e^- \text{ and the strongly interacting particles. For a discussion on the limits of such point interactions, see G. Feinberg and M. Goldhaber, Proc. Natl. Acad. Sci. 45, 1301 (1959).} \]
where \( \langle r^2 \rangle \) is the average of \((\text{radius})^2\) of the “mixed” charge distribution between \( \pi^+ \) and \( \pi^0 \). The decay rate is given by \([\text{neglecting the mass of } \epsilon^0\]

\[
\text{Rate}(\pi^0 \rightarrow \pi^+ + \pi^- + \epsilon^0) = (1272 \pi e^2 / m^2 \langle r^2 \rangle) m \times [1 - (1 - \epsilon^2)](1 - 8\epsilon + \epsilon^2 - 12\epsilon \ln \epsilon),
\]

(34)

where \( \epsilon = (m_{\pi}/m_{\pi'})^2 \).

(35)

The corresponding spectrum of \( \epsilon^0 \) is, apart from a normalization constant, given by

\[
[4k_\pi k_\pi - 2m_{\pi}(k_\pi + k_\pi) + (m_{\pi}^2 - m_{\pi'}^2)] d k_\pi d k_\pi,
\]

(36)

where \( k_{\pi} \) is the energy of \( \pi^0 \). The rate of \( \pi^0 \rightarrow 2\gamma \) may be estimated by using SU(3) symmetry:

\[
\text{Rate}(\pi^0 \rightarrow 2\gamma) = (m_{\pi}/m_{\pi'})^3 \times \text{Rate}(\pi^0 \rightarrow 2\gamma).
\]

(37)

Combining Eqs. (34) and (37), we find the ratio

\[
\frac{\text{Rate}(\pi^0 \rightarrow \pi^+ + \pi^- + \epsilon^0) / \text{Rate}(\pi^0 \rightarrow 2\gamma)}{\sim 0.04 \langle r^2 \rangle m_{\pi'}^2},
\]

(38)

which is \( \sim 1 \) if \( \langle r^2 \rangle \) is set arbitrarily to be the same as the mean-square radius of the proton.

If reaction (29) is observed, then it clearly indicates the \( C \) noninvariance in \( \pi^0 \) decay, and it shows that such violation also does not conserve the isospin \( I \).

Another possibility is to study the spectrum of \( \pi^+ \) and \( \pi^- \) in the decays

\[
\pi^0 \rightarrow \pi^+ + \pi^- + \pi^0
\]

(39)

and

\[
\pi^0 \rightarrow \pi^+ + \pi^- + \gamma.
\]

(40)

Let \( dN(E_\pi, E_\pi) \) be the number of events in which the energies of \( \pi^+ \) and \( \pi^- \) are between \( E_{\pi} \) and \( E_{\pi} + dE_{\pi} \). In either of these two reactions, the observation of

\[
dN(E_\pi = E_\pi, E_\pi = E_\pi) \neq dN(E_\pi = E_\pi, E_\pi = E_\pi)
\]

(41)

for any energies \( E_\pi \) and \( E_\pi \) is an unequivocal proof of \( C \) noninvariance.

A possible \( \pi^\pm \) asymmetry in reaction (39) has been discussed elsewhere, under the assumption that a \( T \) and \( CP \) noninvariant \( H \) exists whose coupling constant is given by Eq. (7). If \( H \) strongly violates \( C \) and \( T \), then \( H \) represents simply the second-order electromagnetic effect. We note that if \( K \) contains only an isoscalar part \( K^s \), then \( H \) satisfies the \( | \Delta I | = 2 \) rule; otherwise, \( H \) may contain a \( | \Delta I | = 2 \) and \( | \Delta I | = 1 \) term.

Any \( \pi^\pm \) asymmetry in reaction (40) must be due to the existence of the \( J = 0 \) (or 2) part of \( K^s \); in addition, there must be strong pion interactions. To analyze such asymmetries, we may make a partial-wave analysis of the \((2\pi)\) system for reaction (40) and the decay

\[
\pi^0 \rightarrow 2\pi^+ + \gamma
\]

(42)

which violates \( C \) conservation. As an illustration, only the lowest possible angular-momentum states will be

\[16^\text{T. D. Lee, Phys. Rev. 139, B1415 (1965).}

kept. The \((2\pi)\) can be either in an \( I = 1 \) state produced by \( J^s \), or in an \( I = 0 \) state produced by \( K^s \). The amplitudes for these two states can be represented, respectively, \( \gamma \mathbf{a}(p \cdot H) \) and \( i\gamma \mathbf{b}(p \cdot H)(p \cdot k) \), where \( p \) is the relative momentum of the two pions, \( k \) is the momentum of the photon, and \( H \) is its magnetic field.

The energy distribution of reaction (40) is, then, given by the invariant phase space times

\[
k^\mu\mathbf{a}(p \cdot k)\mathbf{a}(p \cdot k) \times [a^2 + b^2(p \cdot k)^2 + i(a^2b - b^2a)(p \cdot k)],
\]

(43)

where \( p \) ranges over the entire momentum space and \( k \) is a unit vector along \( k \); the corresponding density for reaction (42) is given by

\[
[b \cdot \mathbf{a}(p \cdot k)\mathbf{a}(p \cdot k)] k^2,
\]

(44)

where \( p \) ranges only over half of the momentum space. From \( CPT \) invariance, we find that the relative phase between \( a \) and \( b \) is given by \((\delta_a - \delta_b) \) or \((\pi + \delta_a - \delta_b) \), where \( \delta \) is the strong-interaction phase shift of the 2-pion system in the \( I = 1 \) state and \( \delta \) is the same in the \( I = 0 \) state. Thus, the asymmetry, if it exists, can also be used to obtain information concerning the pion interactions.

3. Weak Decays

We consider first the \( K^0 \) system. To make our analysis definite, we will assume here that \( H_{\pi k} \) is invariant under both \( T \) and \( CP \), and \( H \), has a large \( C \), \( T \) noninvariant part (i.e., \( K^s \neq 0 \)). To zeroth order in \( \theta_{\pi} \), there is no \( CP \)-violating effect.

Thus,

\[
CP = +1, \quad \text{for } K^0
\]

and

\[
CP = -1, \quad \text{for } K^0.
\]

(45)

If we use Eq. (45) and note that the three pions in the decay \( K^0 \rightarrow 3\pi \) are produced predominantly in \( s \) states, it follows, then, that

\[
(CP)_{s^+} = -1.
\]

(46)

Thus, \( C^s \) is +1 which confirms Eq. (14).

To first order in \( \theta_{\pi} \), since \( H_{\pi k} \) does not conserve \( C \), the decay

\[
K^0 \rightarrow \pi^0 + \epsilon^0 + \epsilon^-
\]

(47)

can occur via either \( J^s \) or \( K^s \), where \( i = 1 \) or 2. For either decay, there is only one form factor, due to current conservation; therefore, it does not yield any direct information concerning possible \( T \) or \( CP \) noninvariance. On the other hand, the decay

\[
K^0 \rightarrow \pi^+ + \pi^- + \gamma
\]

(48)

can be used to test possible \( CP \) noninvariance. Let \( e \cdot \mathbf{M}_{\pi^0}(p, k) \) and \( e \cdot \mathbf{M}_{\pi^+}(p, k) \) be, respectively, the transition amplitudes due to \((J^s A_\pi) H_{\pi k} \) and \((K^s A_\pi) H_{\pi k} \), where \( A_\pi \) is the electromagnetic field operator, \( p \) is the relative momentum between \( \pi^+ \) and \( \pi^- \), and \( k \) is the
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momentum of $\gamma$, and $\varepsilon$ is its polarization vector. By applying $CP$, we find

$$M_{\gamma}(p+k) = \mp M_{\gamma}(p-k)$$

(49)

and

$$M_{\gamma}(p+k) = \mp M_{\gamma}(p-k),$$

(50)

where the upper signs are for $i=1$, and the lower signs for $i=2$. The probability distribution, after summing over the two polarization directions of $\gamma$, is proportional to

$$\sum_{\alpha, \beta} \left[ (M_{\alpha})^* \cdot M_{\beta} - (M_{\beta})^* \cdot M_{\alpha} \right],$$

(51)

where the sum extends over $\alpha$ (and $\beta$) = $J$ and $K$, and $k$ is a unit vector along $k$. By using Eqs. (49) and (50), we find that in expression (51) the interference term between $M_{\alpha}$ and $M_{\beta}$ is an odd function in $k$, and, therefore, it is also an odd function in $p$; the other terms are even in $p$. Thus, the assumption that $K_{\gamma}$=0 can result in a $\pi^\pm$ asymmetry, similar to that given by Eq. (41):

$$dN(E_+ = E_{1+}, E_- = E_{1-}) = \neq dN(E_+ = E_{2+}, E_- = E_{2-}).$$

Conversely, the observation of such an asymmetry gives also a direct proof of $CP$ noninvariance in the $K^\pi$ decays.

To second order in $g_\omega$ decays such as $K^\pi \to 2\pi$ and $K^\mu \to 3\pi^\pm$ can occur. The smallness of the observed rate of reaction (1) is attributed to the smallness of the fine-structure constant. To the same order, explicit $CP$- and $T$-noninvariant effects can also be observed in $K_{\mu}$ and $K_{\pi}$ decays.

For the weak decays of charged mesons, second order in $g_\omega$ there exist explicit $T$-noninvariant observables, such as $\sigma_{\pi} = (p_\pi \times p_\omega)$. The $T$-noninvariant amplitudes are, however, smaller than the corresponding $T$-invariant amplitudes by a factor $\sim O(\alpha)$. In the radiative decay $\pi^\pm$ (or $K^\pm \to \mu^\pm + \nu$) neutrino + $\gamma$, we may consider only the inner-bremstrahlung process. Since both the lepton current and the weak interaction are assumed to be invariant under $T$, no $T$-noninvariant term such as $\sigma_{\pi} = (p_\pi \times p_\omega)$ can be observed.

From $CP$ invariance, we have (neglecting higher order terms in $H_{\omega\omega}$, but valid to all orders in $H_{\omega\omega}$)

$$\sum_{\alpha} \left[ \text{Rate}(K^+ \to n\pi^+ \gamma) + \text{Rate}(K^+ \to n\pi^- \gamma) \right]$$

$$= \sum_{\alpha} \left[ \text{Rate}(K^- \to n\pi^- \gamma) + \text{Rate}(K^- \to n\pi^+ \gamma) \right],$$

(52)

$$\sum_{\alpha} \left[ \text{Rate}(K^+ \to n\pi^+ + n\pi^-) \right.$$

$$+ \text{Rate}(K^+ \to n\pi^- + l^+ + \nu_l) \right.$$

$$= \sum_{\alpha} \left[ \text{Rate}(K^- \to n\pi^- + l^- + \nu_l) \right.$$

$$+ \text{Rate}(K^- \to n\pi^+ + l^- + \nu_l) \right.$$

(53)

$$\text{Rate}(K^+ \to l^+ + \nu_l) + \text{Rate}(K^+ \to l^- + \nu_l + \gamma)$$

$$= \text{Rate}(K^- \to l^- + \nu_l) + \text{Rate}(K^- \to l^+ + \nu_l + \gamma),$$

(54)

where $l = e$ or $\mu$, and $\gamma$ stands for any numbers of photons and $(l^\pm, \nu_l)$ pairs. For decays that are allowed by $H_{\omega\omega}$, from $CP$ invariance, each individual partial decay rate of $K^+$ is equal to that of $K^-$, provided virtual effects of $H_{\omega\omega}$ are neglected. Thus, e.g., neglecting $O(\alpha)$, we obtain

$$\text{Rate}(K^+ \to \pi^+ + 2\pi^0) = \text{Rate}(K^- \to \pi^- + 2\pi^0),$$

(55)

$$\text{Rate}(K^+ \to 2\pi^+ + \pi^0) = \text{Rate}(K^- \to 2\pi^- + \pi^0),$$

(56)

etc. The decay $K^\pm \to \pi^\pm + \pi^0$ is forbidden by $H_{\omega\omega}$ if $K_{\beta}$ rigorously satisfies the $|\Delta I| = 1$ selection rule. In this case, the $K_{\beta}$ decay can only occur through $H_{\omega\omega} + H_{\omega\omega}$, which violates $CP$ invariance. However, by using Eqs. (52)-(56) and the experimental limits on radiative decays, we find that the equality

$$\text{Rate}(K^+ \to \pi^+ + \pi^0) = \text{Rate}(K^- \to \pi^+ + \pi^0)$$

(57)

should hold to an accuracy $\sim O(10^{-3})$. Thus, any lack of difference between the $K_{\beta^+}$ and $K_{\beta^-}$ branching ratios does not reflect the $T$ invariance or $T$ noninvariance of $H_{\gamma}$.

IV. DECAYS OF THE VECTOR MESONS

The vector mesons all decay through $H_{\omega\omega}$. If $H_{\gamma}$ is not invariant under $C$ and $T$, then, through second-order processes, $C$, $T$-noninvariant effects, such as $\pi^\pm$ asymmetry in $\phi^0 \to \pi^\pm + \pi^0$, $\phi^0 \to 2K^\pm$, $\rho^0 \to \pi^\pm + \pi^0$, etc., can occur. The $C$, $T$-noninvariant amplitudes are smaller than the corresponding $C$, $T$-invariant amplitudes by a factor $\sim O(\alpha)$. In the following, we will discuss in detail two types of $C$, $T$-noninvariant processes that can occur to the first order in $g_\omega$.

1. $A^0(1^-) \to B^0(1^-) + \gamma$

In this class, there are

$$\phi^0 \to \omega^0 + \gamma,$$

(58)

and

$$\phi^0 \to \rho^0 + \gamma,$$

(59)

$$\omega^0 \to \rho^0 + \gamma.$$  

(60)

We observe that by using Eq. (45) and the decay $\phi^0 \to K^\pm + K^\pm$, it follows that

$$C_{\phi} = -1.$$  

(61)

From the fact that the $\pi^\pm$, $\pi^\mp$ in the decay $\omega^0 \to \pi^+ + \pi^- + \pi^0$ are produced in an antisymmetric state, or $\omega^0 \to 3\pi^0$, we may conclude

$$C_{\omega} = -C_{\phi} = -1.$$  

(62)

The $C_{\phi}$ is, according to Eq. (26), also $-1$. Decays such as $\omega^0 \to \pi^0 + \gamma$ or $\omega^0 \to \pi^0 + e^+ + e^-$ conserve $C$. However, reactions (58)-(60) all violate $C$ invariance. Thus, only the current $K_{\beta}$ can contribute to these decays. Furthermore, reaction (58) can occur only if $K_{\beta^+} \neq 0$ and reactions (59) and (60) can occur only if $K_{\beta^0} \neq 0$. 

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By using invariance under space reflection and the continuous Lorentz transformations, the general matrix element of $g_\mu$ between any two physical (1→) states $|A\rangle$ and $|B\rangle$ is given by
\begin{equation}
\langle B|g_\mu|A\rangle = i e \varphi \varphi' \left(\lambda' + \lambda\right) \left(F_\lambda g_\mu + F_{\lambda'}g_{\mu'}\right) + F_{\lambda'}g_\mu + F_{\lambda}g_{\mu'} + \varphi_0 \left(F_{\lambda}g_\mu + F_{\lambda'}g_{\mu'}\right),
\end{equation}
where $\lambda_0$, $\lambda_0'$ are, respectively, the 4-momenta of the states $|A\rangle$ and $|B\rangle$, and $\varphi_0 = \lambda_0 - \lambda_0'$. The polarizations of the initial and final states are given by $\varphi_\lambda'$ and $\varphi_\lambda$, respectively, which satisfy the normalization conditions
\begin{equation}
\varphi^2 = (2\omega_\lambda)^{-1} \quad \text{and} \quad \varphi'^2 = (2\omega_{\lambda'})^{-1},
\end{equation}
where $\omega_\lambda$ and $\omega_{\lambda'}$ are the energies of $A$ and $B$. From current conservation, we have
\begin{equation}
(m_\lambda^2 - m^2)F_\lambda + q^2F_\lambda = 0
\end{equation}
and
\begin{equation}
(m_{\lambda'}^2 - m^2)F_{\lambda'} + q^2F_{\lambda'} + F_{\lambda} = 0,
\end{equation}
where $m_\lambda$ and $m_{\lambda'}$ are the masses of $A$ and $B$. For the radiative decay
\begin{equation}
A \to B + \gamma,
\end{equation}
the longitudinal part $\varphi_0 \left(F_{\lambda}g_\mu + F_{\lambda'}g_{\mu'}\right)$ cannot contribute. Assuming that all form factors are regular at $q^2 = 0$ and dropping the longitudinal part, we find, at $q^2 = 0$, $\langle B|g_\mu|A\rangle$ becomes
\begin{equation}
\left(4\lambda + \lambda'\right)g_\mu g_{\mu'} + b \left(\delta_0 \varphi - \delta_0' \varphi'\right) - \frac{1}{2} \left(m_\lambda^2 - m^2\right) \left(\delta_0 \varphi + \delta_0' \varphi'\right) \varphi,
\end{equation}
where $a$ and $b$ are related to these form factors by
\begin{equation}
a = F_2(0)
\end{equation}
and
\begin{equation}
b = \frac{1}{2} \left[F_3(0) - F_4(0)\right].
\end{equation}

If $m_B = m_A = m$, then as $q_0 \to 0$, expression (68) corresponds to a system of two spin-1 neutral particles with a “mixed” magnetic moment $= (eb/2m) \times \text{spin}$ and a “mixed” quadrupole moment $= (e/m^5)(2am^2 - b)$. If $A$ and $B$ are the same particle, then $\langle A|g_\mu|A\rangle = 0$ by CPT invariance and Hermiticity. \] The rate for reaction (67) is given by
\begin{equation}
\begin{aligned}
\text{Rate}(A \to B + \gamma) &= (24)^{-1} \omega m_A - 5m g^2 (m_\lambda^2 + m^2) \\
&\times (m_{\lambda'}^2 - m^2)^2 g^2,
\end{aligned}
\end{equation}
where
\begin{equation}
g^2 = \frac{1}{2} \left(m_\lambda^2 - m^2\right) \left[a^2 + b^2\right] + \frac{1}{2} \left(m_{\lambda'}^2 - m^2\right) \left(a' b + a b'\right).
\end{equation}

If $H_\gamma$ violates $C$, $T$ invariance strongly and if $K^*_L$ exists, then the parameter $g^2$ for $\varphi^2 \to \varphi^2 + \gamma$ may be 1, and the corresponding branching ratio becomes $\approx 1.9\%$. Similarly, if $K^*_L$ exists and if the corresponding $g^2$ is taken to be 1, then the branching ratio for reaction (59) is $\approx 2.4\%$. The reaction (60) has an extremely small branching ratio, so that it is unlikely to be of any practical use.

If $H_\gamma$ does strongly violate $C$, $T$ invariance, observations of reaction (58) and (59), though difficult, may become feasible. The detections of these decay modes are unambiguous proofs of $C$ noninvariance.

2. $A^0(1\to) \to \pi^+ + \pi^- + \gamma$

There are three such decays:
\begin{equation}
\phi^0 \to \pi^+ + \pi^- + \gamma,
\end{equation}
\begin{equation}
\phi^0 \to \pi^+ \pi^- + \gamma,
\end{equation}
and
\begin{equation}
\rho^0 \to \pi^+ + \pi^- + \gamma.
\end{equation}

If $H_\gamma$ is not invariant under $C$, then there would be an asymmetry in the $\pi^+$ and $\pi^-$ distribution in these decays [see Eq. (41)],
\begin{equation}
dN(E_k = E_1, E_\gamma = E_2) \neq dN(E_k = E_2, E_\gamma = E_1).
\end{equation}

Such an asymmetry can occur in reactions (73) and (74) if $K^*_L$ exists, and in reaction (75) if $K^*_S$ exists.

The analysis of these asymmetries is similar to that in the case of $\rho^0$ decay. We make a partial-wave analysis of the $(2\pi)$ system for these reactions together with
\begin{equation}
A^0 \to 2\pi^+ + \gamma,
\end{equation}
where $A^0$ stands for either one of the three mesons: $\phi^0$, $\phi^0$, or $\rho^0$. Reaction (76) does not violate $C$ invariance.

As an illustration of the form of asymmetry that might be observed in these decays, we keep only the lowest possible angular-momentum states. The $(2\pi)$ system can be either in a $l = 0$ state via $J_\mu$, or in a $l = 1$ state via $K_\mu$. The amplitudes for these two states can be represented by $\varepsilon_3\varepsilon_1$ and $i\varepsilon_2\varepsilon_1\varepsilon_1\varepsilon_1\varepsilon_1$, respectively, where $\varepsilon$ is the polarization vector of $A^0$, $p$ is the relative momentum of the two pions, and $E$ and $H$ are, respectively, the electric and magnetic field of the $\gamma$. The complex constants $a$ and $b$ are two phenomenological parameters; they are comparable in magnitude, if the $C$, $T$ violation is a large one. The pion spectrum, after summing and averaging over the polarizations of $A^0$ and $\gamma$, is given by the invariant phase space times
\begin{equation}
k^2 \left(\frac{1}{2} |a|^2 + \frac{1}{2} |b|^2 \left[\gamma^2 + (p\cdot\hat{k})^2\right] - i(a'b - ab') \gamma (p\cdot\hat{k})\right)
\end{equation}
for reaction $A^0 \to \pi^+ + \pi^- + \gamma$, and the same phase space times
\begin{equation}
k^2 \left(\frac{1}{2} |a|^2 \right)
\end{equation}
for reaction (76), where $\hat{k}$ is a unit vector along the momentum direction of $\gamma$. In the evaluation of the phase space $p$ ranges over the entire momentum space in (77), but only over half of the momentum space in (78). From CPT invariance, the relative phase between $a$ and $b$ is given by $i(\delta_2 - \delta_2')$ or $i(\delta_2 - \delta_2' + \pi)$, where $\delta_2$ and $\delta_2'$ are, respectively, the appropriate average phase shifts of the $(2\pi)$ system in the $I = 0$ state and the $I = 1$ state.

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Before leaving the meson systems, we remark that the identical analysis can be applied to the annihilation of the nucleon and antinucleon system at rest; e.g.,

$$p + \bar{p} \rightarrow \pi^+ + \pi^- + \gamma.$$  \hfill (79)

The initial $J^P_S^2$ state has the identical symmetry properties as a mixture of $\rho^0$ and $\pi^0$, and the $I^P_S$ state has the same properties as a mixture of $\phi^0$ (or $\phi^0$) and $\rho^0$.

**V. $\Sigma^0 \rightarrow \Lambda^0 + e^+ + \nu_e$**

For a baryon system, tests of possible $T$ noninvariance in $H$, become more accessible than that of $C$ noninvariance. A suitable case is

$$\Sigma^0 \rightarrow \Lambda^0 + e^+ + \nu_e.$$  \hfill (80)

Let $\sigma_2$, $\sigma_A$, $p$, $k_+$, and $k_-$ denote, respectively, the Pauli spin matrix of $\Sigma$, spin matrix of $\Lambda$, momentum of $\Lambda^0$, momentum of $e^+$, and momentum of $\nu_e$. Since the lepton currents conserve $C$, $P$, and $T$, the distribution, as well as any $T$-noninvariant term, must be symmetric with respect to the exchange between $k_+$ and $k_-$. In the one-photon approximation, we define the normal vector of the decay plane to be parallel to

$$N = \hat{p} \times (k_+ + k_-),$$  \hfill (81)

where $\hat{p}$, $k_+$, and $k_-$ are unit vectors along $p$, $k_+$, and $k_-$. Neglecting the possibility of any spin measurements for $e^+$, the only $T$-noninvariant, but $P$-invariant, observables are

$$\sigma_2 \cdot N$$  and $$\sigma_A \cdot N$$, \hfill (82)

which are even functions with respect to the exchange $e^+ + \nu_e$. If either one of these averages $\langle \sigma_2 \cdot N \rangle$ and $\langle \sigma_A \cdot N \rangle$ is not zero, then it clearly proves $T$ noninvariance, and consequently, also $C$ noninvariance in this decay.

Sometimes, in order not to lose any information contained in the limited statistics available, it may be helpful to use the detailed form of the spin-momentum distribution for an experimental analysis, rather than a simple over-all average. The theoretical results of these distribution functions for reaction (80) are given below.

The matrix element of $\sigma_2$ is given by

$$\langle \Lambda | J_\tau | \Sigma \rangle = \langle \Lambda | J_{\tau} | K_+ K_- | \Sigma \rangle$$

$$= i e U_A \gamma_\tau \{ i \gamma_\tau \gamma_\nu F + i (\Lambda_\alpha + \Sigma_\rho) \} F'$$

$$+ i (\Lambda_\alpha - \Sigma_\rho) F'' \} U_\Sigma,$$  \hfill (83)

where $\Lambda$, $\Sigma$ are, respectively, the 4-momenta of the states $| \Lambda \rangle$ and $| \Sigma \rangle$, $U_\Sigma$ and $U_A$ are spinor solutions of the free-particle Dirac equations with the same 4-momenta as the physical $\Sigma^0$ and $\Lambda^0$. If $T$ invariance holds, then $F$, $F'$, and $F''$ are all real functions of $\varrho^2$, where

$$\varrho^2 = (\Lambda_\alpha - \Sigma_\rho)^2.$$  \hfill (84)

In the following, we will, however, assume these functions to be complex so as to violate $T$ invariance.

From current conservation, these functions satisfy the following relation

$$F = (m_2 + m_4) F' + (m_2 - m_4) \varrho^2 F''$$,  \hfill (85)

where $m_4$ and $m_2$ are the masses of $\Lambda^0$ and $\Sigma^0$, respectively. Thus, there are, in general, only two independent complex form factors. Let $F_0$ and $F'_b$ be the values of $F$ and $F'$ at $\varrho^2 = 0$. We have

$$F_0 = (m_2 + m_4) F'_b$$  \hfill (86)

which may be regarded as the "mixed" gyromagnetic ratio between $\Sigma^0$ and $\Lambda^0$. The value of $F_0$ is related to the decay $\Sigma^0 \rightarrow H^0 + \gamma$:

$$\text{Rate} (\Sigma^0 \rightarrow H^0 + \gamma) = \frac{1}{3} | F_0 |^2 (m_2 - m_4)^3$$

$$\times (m_2 + m_4).$$  \hfill (87)

The initial $\Sigma^0$ may be produced in any reaction, or any combination of reactions. The spin state of $\Sigma^0$ in its rest system is completely characterized by a $(2 \times 2)$ density matrix:

$$D_{\Sigma^0} = \frac{1}{2} (1 + \sigma_2 \cdot s_\Sigma),$$  \hfill (88)

where $s_\Sigma$ is a real vector whose direction and magnitude determine the average spin direction and polarization of the initial $\Sigma^0$.

By using Eqs. (83) and (88), it is straightforward to evaluate the resulting $(2 \times 2)$ density matrix $D_A$ for the $\Lambda^0$. It is convenient to introduce $G$ which is a linear function of $F$ and $F'$:

$$G = (m_4 + m_2) F - 2 m_2 (m_4 + E) F'$$  \hfill (89)

where

$$E = (p^2 + m_\Sigma^2)^{1/2}$$  \hfill (90)

and

$$\varrho = | p |.$$  \hfill (91)

In terms of the form factors $G$ and $F$, the density matrix for the $\Lambda^0$ is given by (neglecting the masses of $e^+$, but without any nonrelativistic approximation)

$$D_A = G^2 (1 + k_+ \cdot k_-) + 2 F F' p [1 - (\hat{p} \cdot k_+)(\hat{p} \cdot k_-)]$$

$$+ i (GF^* - GF) \{ \varrho N \cdot (s_2 + s_A) + D_{\Sigma^0} \},$$  \hfill (92)

where $N$ is defined in Eq. (81). We note that the coefficients of $N \cdot s_2$ and $N \cdot s_A$ are the same, and can be nonzero only if $G$ and $F$ are not relatively real, which violates the condition of $T$ invariance.

The term $D_{\Sigma^0}$ contains all the spin-spin correlations; it is, therefore, zero if we average over either the initial spin direction $s_2$ or the final spin direction of $\Lambda^0$. The detailed form of $D_{\Sigma^0}$ is somewhat complicated, and is given by

$$D_A = \sigma_2 \cdot s_{\Sigma''}.$$  \hfill (93)
\[ S_{A'} = s_2(GG^T(1+\mathbf{k}_+ \cdot \mathbf{k}_-)+2FF^T[\mathbf{p}^2(\mathbf{k}_+ \cdot \mathbf{k}_-)-(\mathbf{p} \cdot \mathbf{k}_+)(\mathbf{p} \cdot \mathbf{k}_-)]) + p[-(GG^T+FG^T)[s_2(\mathbf{k}_+ \cdot \mathbf{k}_-)+2FF^T(-s_2 \mathbf{p})(1+\mathbf{k}_+ \cdot \mathbf{k}_-)+(\mathbf{p} \cdot \mathbf{k}_+)(s_2 \mathbf{k}_-)+(\mathbf{p} \cdot \mathbf{k}_-)s_2 \mathbf{k}_-)] + \mathbf{k}_4\{GG^T+FG^T\}[s_2 \mathbf{p}_2+2FF^T[-p^2(s_2 \mathbf{k}_-)+(\mathbf{p} \cdot \mathbf{k}_-)(\mathbf{p} \cdot \mathbf{s}_2)] + \mathbf{k}_-\{GG^T+FG^T\}[(\mathbf{p} \cdot \mathbf{s}_2)+2FF^T[-p^2(s_2 \mathbf{k}_-)+(\mathbf{p} \cdot \mathbf{k}_-)(\mathbf{p} \cdot \mathbf{s}_2)] \}. \] (94)

Combining Eqs. (92) and (94), we may write
\[ D_A = T + \sigma_A \cdot S_A, \] (95)
where
\[ S_A = i(GG^T - FG^T)\mathbf{p}N + S_{A'} \] (96)
and
\[ T = \frac{1}{2} \text{tr} D_A, \] (97)
which is related to the decay rate by
\[
\text{Rate}(\Sigma^+ \rightarrow \Lambda^0 + e^+ + e^-) = \alpha^2 \int \left[ 4\pi E(m_\Lambda + E) \right]^{-1} (q^2)^{-2} \times d^4k_d d^4k_b \left( k_+ + k_- + E - m_\Sigma \right). \] (98)

In the above, all momenta are measured in the rest system of the \( \Sigma^0 \). Let us first choose the \( z \) axis to be parallel to \( p \), and then make a Lorentz transformation in the \( (t, z) \) subspace to a particular rest system of \( \Lambda \). In this system, the spin direction of \( \Lambda^0 \) is parallel to \( S_\Lambda \) and its magnitude is \( T \cdot s_\Lambda \).

In this decay, the range of \( \| q^2 \| \) is \( \leq (m_\Sigma - m_\Lambda)^2 \); thus, we may expand \( G \) and \( F \) as power series in \( q^2 \). By using
\[ q^2 = 2E(m_\Sigma - (m_\Sigma^2 + m_\Lambda^2)), \] (99)
and Eqs. (86) and (89), we find \( G(q^2 = 0) = 0 \). For small \( q^2 \), \( G \) and \( F \) are given by
\[ F = F_0 + O(q^2), \] (100)
and
\[ G = (dG_0/dq^2)q^2 + O(q^2). \] (101)

Substituting these expressions into Eq. (92), we find that if \( s_2 = 0 \), the approximate amount of \( \Lambda^0 \) polarization along \( N \) is given by
\[ 2 \sin^2 \theta \rho \left( F_0^{-1} \frac{dG_0}{dq^2} \right) \times \frac{k_+ k_- (k_+ - k_-) \sin \theta}{k_+^2 + k_-^2 - k_+ k_- (1 - \cos \theta)}, \] (102)
where \( k_\pm = |k_\pm| \), \( \theta \) is the angle between \( k_+ \) and \( k_- \),
\[ \cos \theta = \left( \frac{2k_+ k_-}{k_+ - k_-} \right)^{-1}, \] (103)
\[ \phi \] is the relative phase between the two parameters \( (dG_0/dq^2) \rho \) and \( F_0 \), and \( \phi = 0 \) or \( \pi \) if \( T \) invariance holds. The dimension of \( [(dG_0/dq^2) \rho] / F_0 \) is the same as \( m_\Sigma^{-4} \).

Assuming that \( H_\gamma \) strongly violates \( T \) invariance, and using only simple dimensional considerations, we may expect an over-all average of \( \langle N \cdot \sigma_A \rangle \) to be about
\[ \frac{m_\gamma (m_\Sigma - m_\Lambda)^2}{\ln [(m_\Sigma - m_\Lambda)^2/q_{\min}^2]}, \] (104)
where \( q_{\min}^{-2} \) is the minimum value of \( q^2 \) among the events and we have taken \( (dG_0/dq^2) \rho \sim \frac{1}{2} I_\rho (r^2)(m_\Sigma + m_\Lambda) \). Clearly, it is more favorable to take events with relatively large \( q^2 \), but this must be balanced against the small number of such events.

VI. COMPARISON BETWEEN BRANCHING RATIOS OF PARTICLES AND ANTIPARTICLES

We consider first the weak decays, and assume that \( H_\lambda \) conserves \( CP \), but \( H_\gamma \) can have strong violations of \( C \) and \( T \). Thus, the partial weak-decay rates of any particle are the same as that of its antiparticle, provided \( H_\gamma \) is neglected. For example, the equality
\[ \text{Rate}(\Sigma^+ \rightarrow n + \pi^+) = \text{Rate}(\Sigma^- \rightarrow \bar{n} + \pi^-) \] (105)
holds to \( O(\alpha) \). Violation of the equality, Eq. (105), depends not only on \( H_\gamma \), being noninvariant under \( C \) and \( T \), but also on the strong interactions in the final states. A more sensitive test is to compare the partial radiative decay rates. If \( H_\gamma \) violates \( C \), \( T \) invariance strongly, then the radiative decay rate of a particle may be quite different from that of its antiparticle, provided the radiation is not just the inner-bremstrahlung of the lepton currents. The observation of, say,
\[ \text{Rate}(\Sigma^+ \rightarrow n + \pi^+ + \gamma) \neq \text{Rate}(\Sigma^- \rightarrow \bar{n} + \pi^- + \gamma) \] (106)
is a conclusive proof of \( CP \) violation.

These considerations can be applied to any weak decays, and the particular case of \( K \)-meson decays has already been studied in Sec. III.3.

Next, we make a few brief remarks concerning strong reactions. The strong interaction \( H_\alpha \) conserves \( C, P, T \) separately. Since \( H_\gamma \) may be strongly noninvariant under \( C, T \), the differential cross section of
\[ A + B \rightarrow \gamma + C + D + \cdots \] (107)
can be very different from that of its \( C \)-conjugate process
\[ A + B \rightarrow \gamma + \bar{C} + \bar{D} + \cdots, \] (108)
where \( A, B, C, D \cdots \) are any strongly interacting particle states and \( A, B, \cdots \) are the corresponding antiparticle states. The most convenient case is probably
the annihilation of $\rho$ by $\bar{\rho}$; i.e.,

$$A = \rho \quad \text{and} \quad B = \bar{\rho}. \quad (109)$$

Another possible test is to compare the branching ratios, or detailed distributions, of the radiative decays of any resonance with that of its $C$-conjugate state. If $H_\gamma$ violates $C$, $T$ invariance, then the distribution and branching ratio of, say,

$$N^\gamma \rightarrow \rho + \pi^- + \gamma \quad (110)$$

may be different from that of

$$N^{\gamma_0} \rightarrow \bar{\rho} + \pi^+ + \gamma. \quad (111)$$

From $CPT$ invariance, such a difference can occur only if the strong final-state interaction is not neglected.

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AN ELEMENTARY DISCUSSION OF POSSIBLE NON-INVARIANCE UNDER T, CP AND CPT IN HYPERON DECAYS*

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1. INTRODUCTION

The question whether the weak interactions are invariant under the time reversal operation \( T \), or \( CP \) the product of the charge conjugation \( C \) and the space inversion \( P \), has been raised [1] in the early days when the possibility of non-conservation of parity was being studied. After the discoveries [2, 3] that parity is not conserved, several experiments were performed to test the time reversal invariance in weak interactions. It was found that within the experimental accuracy [4], of about a few \( \% \) in relative amplitudes, time reversal invariance holds in the \( \beta \)-decay, and to a much lesser accuracy, the same holds [5] for the \( \Lambda^0 \) decay. If \( CPT \) invariance [6] is assumed, to the same degree of experimental accuracy \( CP \) is also conserved in these decays.

Recently, Christenson et al. observed [7] that the long-lived component of the neutral \( K^0 \) meson can decay into \( (\pi^+ + \pi^-) \), thus suggesting that \( CP \) invariance is violated in the \( K^0 \) decay. The observed non-invariant amplitude is quite small, being only \( \sim 2 \times 10^{-3} \) relative to the corresponding \( CP \) conserving amplitude. If \( CPT \) invariance is assumed, then the same experiment implies that time reversal invariance is also violated.

The experiments which established parity non-conservation usually consist of directly observing a right-left asymmetric effect from an, otherwise, initially right-left symmetric state. The conclusions that space inversion symmetry is violated in these experiments can be reached without any theoretical assumptions. The same is also true for the violation of charge conjugation symmetry. It is important to note

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that in all these weak interaction experiments, which pertain to testing the
time reversal invariance or non-invariance, not a single one consists of
comparing a reaction with its time reversal process. The relations
between these experimental observations and time reversal symmetry
are obtained through indirect theoretical reasoning, and some of these
conclusions are valid only under additional assumptions such as CPT
invariance. Similar criticism applies also to many of the existing tests
of CP invariance.

It seems, therefore, desirable to review the underlying theoretical
arguments of some of these tests, and to separate out the various im-
lications of different symmetry requirements. With this motive, we
will analyze in this note the simple example of the decay of a spin $\frac{1}{2}$
hyperon, say,

$$\Lambda^0 \rightarrow N + \pi$$  \hspace{1cm} (1)

where N stands for either p or n and $\pi$ represents the corresponding
$\pi^-$ or $\pi^0$. The consequences of possible non-invariance under $T$, $CP$
and CPT are derived in Section II. As is well known, the time reversal
invariance in the hyperon decay means [8] that the relative phase
between the final $s_\pm$ and $p_\pm$ amplitudes is determined by the corre-
sponding strong interaction phase shifts. In Section III, the same result
is obtained by an alternative proof which is based only on the recipro-
city relation between different reaction rates, without the explicit use
of the time reversal operator [9] $T$. A simple example is given in Section
IV which illustrates the difference between the consequences of time
reversal invariance in quantum mechanics and that in classical me-
chanics, and which emphasizes again that the present tests of time
reversal invariance concern only the reciprocity relations between
various differential cross-sections, rather than the detailed time re-
versal operation $T$.

Throughout these discussions we assume that the amplitude of
reaction (1) can be represented by the corresponding matrix element
of a Hermitian operator [10] $H_{\text{weak}}$. The validity of the local field
theory, or CPT invariance, is not assumed.

2. $\Lambda^0$ DECAY

In the decay of $\Lambda^0$, the final $(N + \pi)$ system can be in either a $s_\pm$ or a
non-invariance in hyperon decays

\( p_4 \) spin-orbital state. Let these two transition amplitudes be denoted by \( A_s(I) \) and \( A_p(I) \) where \( I = \frac{1}{2} \), or \( \frac{3}{2} \), is the total iso-spin of the final state. The relative phase \( \phi(I) \) is given by

\[
\frac{A_s(I)}{A_p(I)} = \frac{|A_s(I)|}{|A_p(I)|} e^{i\phi(I)}.
\] (2)

Similarly, in the decay of the anti-lambda,

\[
\Lambda^0 \rightarrow N + \pi,
\] (3)

the corresponding \( s_4 \) and \( p_4 \) transition amplitudes are \( \bar{A}_s(I) \) and \( \bar{A}_p(I) \), and their relative phase \( \bar{\phi}(I) \) is given by

\[
\frac{\bar{A}_s(I)}{\bar{A}_p(I)} = \frac{|\bar{A}_s(I)|}{|\bar{A}_p(I)|} e^{i\bar{\phi}(I)}.
\] (4)

The following theorem states the separate consequences of the invariance requirements under \( T \), \( CP \) and \( CPT \) for the \( \Lambda^0 \) and \( \bar{\Lambda}^0 \) decays. Throughout the present paper, we neglect the effects of electromagnetic interactions and assume that the strong interaction is separately invariant under \( T \), \( C \) and \( P \).

**Theorem**

1. If \( T \) invariance holds then, independent of \( CP \) invariance,

\[
\phi(I) = \begin{cases} 
\delta_s(I) - \delta_p(I), & \text{or} \\
\delta_s(I) - \delta_p(I) - \pi 
\end{cases}
\] (5)

and

\[
\bar{\phi}(I) = \begin{cases} 
\delta_s(I) - \delta_p(I), & \text{or} \\
\delta_s(I) - \delta_p(I) - \pi 
\end{cases}
\] (6)

where \( \delta_s(I) \) and \( \delta_p(I) \) are, respectively, the \( s_4 \) and \( p_4 \) phase shifts due to the strong interactions of the \((N+\pi)\) system with a total iso-spin \( I \).

2. If \( CP \) invariance holds then, independent of \( T \) invariance,

\[
A_s(I) = -\bar{A}_s(I)
\] (7)

\[
A_p(I) = +\bar{A}_p(I)
\] (8)

and, consequently,

\[
\phi(I) = \bar{\phi}(I) + \pi.
\] (9)
For convenience, we chose the anti-particle states $\bar{\Lambda}^0$ and $\bar{N}$ to be identically related to their respective particle states $\Lambda^0$ and $N$ through the $CP$ operation.

3. If $CPT$ invariance holds then, independent of either $T$ invariance or $CP$ invariance,

$$|A_s(I)| = |\bar{A}_s(I)|,$$  \hfill (10)

$$|A_p(I)| = |\bar{A}_p(I)|,$$  \hfill (11)

and

$$\frac{1}{2} [\Phi(I) + \bar{\Phi}(I)] = \begin{cases} [\delta_s(I) - \delta_p(I)] + \frac{1}{2}\pi, & \text{or} \\ [\delta_s(I) - \delta_p(I)] - \frac{1}{2}\pi. \end{cases}$$  \hfill (12)

Some of these results, e.g. Eqs. (5) and (6), are well known and have already been proved in the literature [8]. For pedagogical reasons, a formal proof of this theorem is given below.

**Proof.** We consider the rest system of $\Lambda^0$. Let $|(N\pi)^{t,s}_I\rangle$ and $|(N\pi)^{t,p}_I\rangle$ be, respectively, the stationary wave eigen-states of the strongly interacting $(N+\pi)$ system in the $s_+ \text{ and } p_+$ orbits and with a total iso-spin $I = \frac{1}{2} \text{ or } \frac{3}{2}$. The corresponding incoming wave states $|(N\pi)^{in}_I\rangle$ and $|(N\pi)^{in}_I\rangle$ are related to these stationary states by

$$|(N\pi)^{in}_I\rangle = e^{-i\phi(I)}|(N\pi)^{t}_I\rangle,$$  \hfill (13)

where $I = s \text{ or } p$. The transition amplitude $A_s(I)$ is given by

$$A_s(I) = \langle(N\pi)^{in}_I|H_{\text{weak}}|\Lambda^0\rangle$$  \hfill (14)

$$= e^{+i\phi(I)}\langle(N\pi)^{in}_I|H_{\text{weak}}|\Lambda^0\rangle$$  \hfill (15)

where $|\Lambda^0\rangle$ is the physical $\Lambda^0$ state. The time reversal operator $T$ is represented [11] by the joint operation of a complex conjugation times a unitary operator $U_T$. Since the strong interaction is invariant under $T$, all its stationary eigen-states $|j, m\rangle$ which have zero total momentum can be chosen to transform under $T$ as

$$T|j, m\rangle = U_T|j, m\rangle^* = (-1)^m|j, -m\rangle,$$  \hfill (16)

where $j$ is the total angular momentum quantum number and $m$ its $z$-component. Both $|\Lambda^0\rangle$ and $|(N\pi)^{in}_I\rangle$ satisfy Eq. (16). If $H_{\text{weak}}$ is invariant under the time reversal operation, then

$$TH_{\text{weak}}T^{-1} = U_TH_{\text{weak}}U_T^* = H_{\text{weak}},$$  \hfill (17)
and, as a consequence, \( \langle (N\pi)_{l}\,|H_{\text{weak}}|\Lambda^{0}\rangle \) are real. Thus, Eq. (5) can be obtained by using Eq. (15).

For the decay of \( \Lambda^{0} \), we may denote the corresponding incoming wave eigen-state of the strongly interacting \((N + \pi)\) system by \(|(N\pi)_{l}\,\rangle\). Since the strong interaction is invariant under C, Eq. (13) implies that the \(|(N\pi)_{l}\,\rangle\) state is also related to the stationary state \(|(N\pi)_{l}\,\rangle\) by

\[
|\langle N\pi \rangle_{l}\,\rangle = e^{-i\lambda(I)}|\langle N\pi \rangle_{l}\,\rangle.
\]  

Equation (6) can be derived by using the relation

\[
\langle N\pi \rangle_{l}\,\rangle = \langle (N\pi)_{l}\,|H_{\text{weak}}|\Lambda^{0}\rangle.
\]  

To establish the consequences of \( CP \) invariance, we may choose

\[
|\Lambda^{0}\rangle = CP|A^{0}\rangle,
\]

\[
|\langle N\pi \rangle_{l}\,\rangle = +CP|\langle N\pi \rangle_{l}\,\rangle
\]

and

\[
|\langle N\pi \rangle_{l}\,\rangle = -CP|\langle N\pi \rangle_{l}\,\rangle
\]

where \( \alpha \) = stationary or incoming. Equations (7)–(9) are the direct consequences of the assumption that \( H_{\text{weak}} \) is invariant under \( CP \), i.e.

\[
CPH_{\text{weak}}P^{-1}C^{-1} = H_{\text{weak}}.
\]

If \( H_{\text{weak}} \) is invariant under \( CPT \), then

\[
CPTH_{\text{weak}}T^{-1}P^{-1}C^{-1} = H_{\text{weak}}.
\]

Equations (10), (11) and (12) follow immediately by using Eqs (14)–(16) and (18)–(22). A special consequence of Eqs (10) and (11) is that \( CPT \) invariance implies [1] the equality of life time between \( \Lambda^{0} \) and \( \bar{\Lambda}^{0} \).

We note that Eqs (5) and (6) are consequences of Eqs (7)–(12) [i.e. \( T \) invariance is a consequence of \( CP \) invariance and \( CPT \) invariance], Eqs (7)–(9) are consequences of Eqs (5), (6) and (10)–(12) [i.e. \( T \) invariance and \( CPT \) invariance imply \( CP \) invariance], and that Eqs (10)–(12) are consequences of Eqs (5)–(9) [i.e. \( T \) invariance and \( CP \) invariance imply \( CPT \) invariances].

The absolute magnitudes and the relative phases of these transition amplitudes can be directly measured by studying the decay rates and
the spin directions \([5, 12]\) for reactions (1) and (3). As shown in \([12]\), if in reaction (1) the initial \(A^0\) is at rest and is completely polarized along the unit vector \(\hat{S}_A\), then at any given momentum direction \(\hat{k}\) the final nucleon is also completely polarized, and its spin direction \(\hat{S}_N\) (measured in its own rest system) is given by

\[
\hat{S}_N = [1 - \alpha \cos \theta]^{-1}[(-\alpha + \cos \theta)\hat{k} + \beta(\hat{k} \times \hat{S}_A) + \gamma(\hat{k} \times \hat{S}_A) \times \hat{k}]
\]  

(25)

where \(\hat{k}\) and \(\hat{S}_N\) are unit vectors, \(\cos \theta = \hat{k} \cdot \hat{S}_A\),

\[
\alpha = 2 \text{Re} \left( A_s^* A_p \right) / [|A_s|^2 + |A_p|^2],
\]

(26)

\[
\beta = -2 \text{Im} \left( A_s^* A_p \right) / [|A_s|^2 + |A_p|^2],
\]

(27)

and

\[
\gamma = [ |A_s|^2 - |A_p|^2 ] / [ |A_s|^2 + |A_p|^2 ].
\]

(28)

The \(A_s\) and \(A_p\) are, respectively, the corresponding \(s_\pm\) and \(p_\pm\) amplitudes which are linearly related to the \(A_s(I)\) and \(A_p(I)\) by using the appropriate Clebsch-Gordan coefficients depending on whether the nucleon is \(p\) or \(n\). The measurements of the rates and the parameters \(\alpha, \beta, \gamma\) for the decays of \(A^0\) and \(A^0\) give direct tests of \(T\), or \(CP\), or \(CPT\) invariance in these reactions.

Among these tests, the ones for \(CP\) invariance consist of directly comparing two \(CP\) conjugate processes; their physical implications are, therefore, self-evident. The tests for \(T\) invariance and \(CPT\) invariance are less intuitively obvious. For this reason, an alternative proof of Eqs (5), (6) and (10)–(12), based on reciprocity relations, will be given in the next section.

3. RECIPROCITY

In order to make clear the consequence of time reversal symmetry, reaction (1)

\[ A^0 \rightarrow N + \pi \]

should be considered together with its reversed process

\[ N + \pi \rightarrow A^0. \]

(29)

Let \(\langle \hat{k}, \hat{S}_N | M | \hat{S}_A \rangle\) and \(\langle \hat{S}_A' | M | \hat{k}' \rangle\) be, respectively, the transition matrix elements of reactions (1) and (29) where \(\hat{S}_A, \hat{S}_N, \hat{k}\) are the same unit vectors as those used in Eq. (25) and \(\hat{S}_A', \hat{S}_N', \hat{k}'\) are the corresponding unit vectors for reaction (29).
If time reversal symmetry holds, then reaction rates of (1) and (29) are related by the reciprocity relation which states that for arbitrary directions \( \vec{k}, \hat{S}_N \) and \( \hat{S}_A \),

\[
\langle \vec{k}, \hat{S}_N | M | \hat{S}_A \rangle = \langle -\vec{k}_A | M | -\vec{k}, -\hat{S}_N \rangle.
\]  

(30)

This reciprocity relation deals directly with observations. It is important to note that the usual \( T \) invariance implies not only the transition probabilities but also the transition amplitudes satisfying the reciprocity relation [13, 14]. In this section, we will show that the previously proved consequences of \( T \) invariance can be derived by using only the reciprocity relations between the relevant reaction rates.

The reciprocity relation, Eq. (30), holds for the \((N+\pi)\) system in any isotopic spin state \( I \). To first order in \( H_{\text{weak}} \) (but all orders in the strong interaction), the transition matrix elements for reactions (1) and (29) are given, respectively, by

\[
\langle \vec{k}, \hat{S}_N | M | \hat{S}_A \rangle = \langle (\vec{k}, \hat{S}_N)^{\text{in}} | H_{\text{weak}} | \hat{S}_A \rangle
\]

and

\[
\langle \vec{k}_A | M | \hat{S}_N, \hat{S}_A \rangle = \langle (\vec{k}_A, \hat{S}_N)^{\text{out}} | H_{\text{weak}} | (\vec{k}, \hat{S}_N)^{\text{in}} \rangle
\]

where \( |\hat{S}_A\rangle \) and \( |\hat{S}_N\rangle \) are the same physical \( |A^0\rangle \) state used in Eq. (14), but with the \( A^0 \) spin polarized along \( \hat{S}_A \) and \( \hat{S}_N \) respectively. The \( |(\vec{k}, \hat{S}_N)^{\text{out}}\rangle, \) (or \( |(\vec{k}, \hat{S}_N)^{\text{in}}\rangle \)) state is the outgoing (or incoming) eigenstate of the strong interaction for the \((N+\pi)\) system and, in the coordinate representation, it has an asymptotic form that consists of a plane wave with momentum \( k \) and spin \( \hat{S}_N \) plus the appropriate outgoing (or incoming) waves. These eigen-states can be expanded in terms of the spherical waves used in the previous section. For example, in the non-relativistic limit, the explicit asymptotic forms of these expansions in the coordinate representation are given by

\[
\langle r | (\vec{k}, \hat{S}_N)^{\text{out(in)}} \rangle \to e^{\pm i\delta_A(I)} U_N(kr)^{-1} \sin [kr + \delta_A(I)] +
\]

\[
+ ie^{\pm i\delta_A(I)} (\sigma \cdot r)(\sigma \cdot \vec{k}) U_N(kr^2)^{-1} \sin [kr - \frac{1}{2} \pi + \delta_p(I)] + \ldots
\]

(33)

as the relative distance \( r = |r| \to \infty \), where the + signs in the exponents are for the outgoing state and the − signs are for the incoming state. The components of the vector \( \sigma \) are the usual Pauli
matrices, and $U_N$ is a Pauli spinor which satisfies

$$(\sigma \cdot \hat{S}_N)U_N = U_N. \tag{34}$$

Equation (31) and the rotational symmetry property of $H_{\text{weak}}$ allow us to write

$$\langle \hat{k}, \hat{S}_N | M | \hat{S}_A \rangle = U_A^\dagger [a_s(I)e^{i\theta_s(I)} + a_p(I)e^{i\theta_p(I)}(\sigma \cdot \hat{k})] U_A \tag{35}$$

where $\dagger$ denotes the Hermitian conjugation and the spinor $U_A^*$ satisfies

$$(\sigma \cdot \hat{S}_A)U_A = U_A. \tag{36}$$

The $a_s(I)$ and $a_p(I)$ are related to the $A_s(I)$ and $A_p(I)$ of the previous section by

$$A_I = a(I)e^{i\theta(I)} \tag{37}$$

where $l = s$ or $p$. By using the assumed Hermiticity property of $H_{\text{weak}}$, the explicit form of $|\langle \hat{k}, \hat{S}_N \rangle^\dagger \rangle$ and Eq. (32), we find that the corresponding matrix element for reaction (29) is given by

$$\langle \hat{S}_A^\prime | M | \hat{k}', \hat{S}_N \rangle = U_A^\prime \dagger [a_s^* (I)e^{i\theta_s(I)} + a_p^* (I)e^{i\theta_p(I)}(\sigma \cdot \hat{k}')] U_A^\prime \tag{38}$$

where $U_N$ and $U_A$ are the spinors whose spin directions are $\hat{S}_N$ and $\hat{S}_A$, respectively. Substituting Eqs (35) and (38) into Eq. (30), we find that if the reciprocity relation is satisfied then

$$\frac{a_p^*(I)}{a_p(I)} = \frac{a_s^*(I)}{a_s(I)} \tag{39}$$

which gives Eq. (5).

Equations (35) and (38) also determine directly the transition amplitude of the resonant scattering

$$N + \pi \rightarrow A^0 \rightarrow N + \pi. \tag{40}$$

At the resonant energy, the amplitude of (40) is proportional to

$$\langle \hat{k}, \hat{S}_N | M | \hat{k}', \hat{S}_N \rangle = U_N^\dagger [a_s e^{i\theta_s} + a_p e^{i\theta_p}(\sigma \cdot \hat{k})][a_s^* e^{i\theta_s} + a_p^* e^{i\theta_p}(\sigma \cdot \hat{k}')] U_N \tag{41}$$

where $\hat{S}_N$ and $\hat{k}'$ are, respectively, the spin and momentum directions of the initial nucleon and $\hat{S}_N$, $\hat{k}$ are that of the final nucleon. In Eq. (41), we suppress the explicit $I$-dependence in $a_s$ and $a_p$. The reciprocity relation between the transition probabilities for the resonant scattering
is given by
\[ |\langle \hat{k}, \hat{S}_N|M|\hat{k}', \hat{S}_N' \rangle| = |\langle -\hat{k}', -\hat{S}_N|M|\hat{k}, -\hat{S}_N \rangle|, \]
(42)
which can also be used to derive Eq. (39), or Eq. (5). For example, let us consider the simple case of a backward resonant scattering, i.e.
\[ \hat{k} = -\hat{k}'. \]
(43)
Equation (41) becomes simply
\[ \langle \hat{k}, \hat{S}_N|M|\hat{k}, \hat{S}_N' \rangle = U_N'[C + D(\sigma \cdot \hat{k})] U_N', \]
(44)
where
\[ C = |a_s|^2 e^{2i\delta_s} - |a_p|^2 e^{2i\delta_p} \]
and
\[ D = (a_s^* a_p - a_p^* a_s) e^{i(\delta_s + \delta_p)}. \]
(45) (46)
The transition probability for the resonant scattering from, say, \( \hat{S}_N' = \hat{k} \) to \( \hat{S}_N = \hat{k} \) is proportional to \( (C + D) \), and the corresponding probability for the reversed process from \( \hat{S}_N' = -\hat{k} \) to \( \hat{S}_N = -\hat{k} \) is proportional to \( (C - D) \). Thus, if the reciprocity relation, Eq. (42), holds, \( D \) must be zero, which gives another derivation of Eq. (5).

In the same way, by comparing the reaction rates between (3) and
\[ N + \pi \rightarrow \Lambda^0, \]
(47)
we can derive Eq. (6) without explicitly using the anti-unitary operator \( T \); similarly, Eqs (10)–(12) can be derived by comparing the reaction rates between (1) and (47).

In this simple case of \( \Lambda^0 \) and \( \bar{\Lambda}^0 \) decays, we have shown that all the consequences of \( T \) invariance and \( CPT \) invariance can be derived by using only reciprocity relations between the various differential cross-sections. The same can also be established for other presently proposed tests of \( T \) invariance and \( CPT \) invariance in weak interactions [15].

4. DISCUSSIONS

In the decay of \( \Lambda^0 \rightarrow N + \pi \), if the initial \( \Lambda^0 \) is completely polarized along \( \hat{S}_A \), then at any given momentum direction \( \hat{k} \) the final nucleon must also be completely polarized along \( \hat{S}_N \) which is given by Eq. (25).

Let us now consider the reversed reaction \( N + \pi \rightarrow \Lambda^0 \), where the
initial polarization direction $\mathbf{S}_N$ and the incident momentum direction $\mathbf{k}'$ are given by

$$\mathbf{S}_N' = -\mathbf{S}_N \quad \text{and} \quad \mathbf{k}' = -\mathbf{k}. \quad (48)$$

We note that had the system obeyed classical mechanics, then time reversal invariance would imply that the final $A^0$ in the reversed reaction must be completely polarized along the reversed direction $\mathbf{S}_A'$ where

$$\mathbf{S}_A' = -\mathbf{S}_A. \quad (49)$$

For the quantum mechanical system, while the final $A^0$ does remain completely polarized, its direction $\mathbf{S}_A'$ is, in general, different from $-\mathbf{S}_A$.

To demonstrate this, we may consider the special case

$$A_s(I) = -A_p(I) \quad (50)$$

and neglect $\delta_s(I)$ and $\delta_p(I)$. Eq. (5) is, then, satisfied. The final nucleon in the decay $A^0 \to N + \pi$ is now always polarized along $\mathbf{S}_N = -\mathbf{k}$ while the final $A^0$ in the reversed reaction $N + \pi \to A^0$ is always polarized along $\mathbf{S}_A' = -\mathbf{k}'$ which can be very different from $-\mathbf{S}_A$.

The time reversal operator $T$ in quantum mechanics relates the solution of the Schrödinger Equation $\psi(t)$ at a time $t$ with that at $-t$. The final state $\psi(t = \infty)$ is a coherent mixture of $s_+ \text{ and } p_\pm$ waves which, of course, can also be expanded as another coherent mixture of $|\langle \mathbf{k}, \mathbf{S}_N \rangle_{in} \rangle$ states. Under $T$, the state $|\langle \mathbf{k}, \mathbf{S}_N \rangle_{in} \rangle$ becomes $|\langle -\mathbf{k}, -\mathbf{S}_N \rangle_{out} \rangle$, and $T\psi(t = \infty)$ becomes a corresponding coherent mixture of $|\langle -\mathbf{k}, -\mathbf{S}_N \rangle_{out} \rangle$ states which is, obviously, very different from a single $|\langle -\mathbf{k}, -\mathbf{S}_N \rangle_{out} \rangle$ state. However, as stated in Eq. (48), it is precisely this single $|\langle -\mathbf{k}, -\mathbf{S}_N \rangle_{out} \rangle$ state that is being used as the initial state in the reversed reaction $N + \pi \to A^0$. On the other hand, the reciprocity relation, Eq. (30), does equate the transition probabilities between the $A^0$ decay and the reversed reaction whose initial state is given by Eq. (48).

The above simple example merely illustrates once again these elementary aspects of quantum mechanics. It also illustrates that while the mathematical operation of the anti-unitary operator $T$ deals with the symmetry between the solution $\psi$ at a time $t$ and that
at \(-t\), the direct experimental test of such symmetry properties usually does not go beyond the reciprocity relations between various reaction rates. In connection with the recent observation [7] of Christenson et al., while we can at least envisage theoretically the possibility that, in some distant future, it may become possible to test directly the relevant reciprocity relations [16], it seems virtually impossible to ever construct the desired coherent time reversed state \(T\psi(t = \infty)\) for a direct testing of the symmetry (or, violation of symmetry) properties of the time reversal operation.

I wish to thank Professors G. Feinberg, R. Serber and G. C. Wick for several enjoyable discussions.

REFERENCES

8) T. D. Lee and C. N. Yang, Elementary Particles and Weak Interactions, (Brookhaven National Laboratory, 1957), p. 34.
9) The usual \(T\) invariance preserves the magnitude \(\langle \psi|\phi \rangle = |T\psi|T\phi\rangle\) for all \(\psi\) and \(\phi\) in the Hilbert space, while in this note the reciprocity relation between various reaction rates refers specifically only to those \(\psi\) and \(\phi\) which represent asymptotically the appropriate initial and final systems in which every particle has a definite momentum and a definite spin. Thus, by itself, the \(T\) invariance appears to be a stronger mathematical condition.
10) If, as proposed by Lee and Wolfenstein (Phys. Rev. to be published), the coupling constant of the \(T\) non-invariant interaction, called \(H_F\), is \(\sim 10^4\) times that of the usual \(T\) invariant weak interaction, called \(H_G\), then the relevant operator
$H_{\text{weak}}$ should be the sum of $H_G$ plus the second order term due to $H_F H_G$; otherwise, $H_{\text{weak}} = H_G + H_F$.


15) The mass equality of p and $\bar{p}$, which is valid to all orders in $H_{\text{weak}}$ if CPT invariance holds, can be derived by using the reciprocity relation between, say, $\gamma + p \rightarrow \gamma + p$ and its CPT conjugate process $\gamma + \bar{p} \rightarrow \gamma + \bar{p}$. The same reciprocity relation leads also to the well known identities between the electromagnetic properties of p and $\bar{p}$.

16) Such tests may become feasible in the immediate future if the coupling constant $F$ of the T non-invariant interaction $H_F$ turns out to be $\sim 10^8$ times the Fermi coupling constant $G$ of the usual weak interaction. [See reference [10]]. In such a case, all strong reactions can violate the reciprocity relation by a fractional difference $\sim (10^{-4} - 10^{-5})$ between the relevant reaction rates. The current experimental accuracy of reciprocity relation in strong interactions is $\sim 2 \%$ as determined by L. Rosen and J. E. Brolley, Jr., Phys. Rev. Letters 2 (1959) 98 for the reactions $p + t \rightarrow d + d$. This accuracy is compatible with $F \sim 10^8 G$, but it also implies that $F$ cannot be much bigger than $10^8 G$. [Cf., however, the discussion by J. Prentki and M. Veltman, Physics Letters 15 (1965) 88 in which a different view is attempted.]

Preludes in Theoretical Physics
North-Holland Publishing Co. (1966), pp. 5–16
A talk given at the Oxford International Conference on Elementary Particles,
September, 1965.

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This talk will be divided into two parts: the first part consists of a review of the present status of weak interactions and the second part discusses some recent theoretical speculations about $C$, $P$, $T$ non-invariances.

I. Weak Interactions

All known weak interactions can be divided into three groups:

$$\mathcal{L}_{\text{wk}} = \mathcal{L}_{\ell\ell} + \mathcal{L}_{\ell h} + \mathcal{L}_{hh}$$  \hspace{1cm} (1)

where the subscript $\ell$ denotes the leptons and $h$ the hadrons. The three terms \( \mathcal{L}_{\ell\ell} \), \( \mathcal{L}_{\ell h} \) and \( \mathcal{L}_{hh} \) describe, respectively, the weak reactions which involve only leptons, leptons and hadrons, and only hadrons. Among these, the $\mu$-decay part of \( \mathcal{L}_{\ell\ell} \) is known explicitly and is given by

$$\left( \mathcal{L}_{\ell\ell} \right)_{\mu-\text{decay}} = \frac{g_\mu}{\sqrt{2}} \left( j_\lambda \right)_\mu^* \left( j_\lambda \right)_e + \text{h.c.}$$  \hspace{1cm} (2)

where

$$\left( j_\lambda \right)_\ell = i \Psi_\ell \gamma_4 \gamma_\lambda \left( 1 + \gamma_5 \right) \Psi_\ell ,$$  \hspace{1cm} (3)

$\ell = e$ or $\mu$, and \( \left( j_\lambda \right)_\ell^* \) is the hermitian conjugate of \( \left( j_\lambda \right)_\ell \) times +1, or -1, depending on whether $\lambda \neq 4$, or $\lambda = 4$.

This (current $X$ current) form of $(V, A)$ interaction\(^1\,^2\) can be generalized to include all leptonic parts of the weak interaction:

$$\frac{g_\mu}{\sqrt{2}} \left( J_\lambda^* J_\lambda \right) = \mathcal{L}_{\ell\ell} + \mathcal{L}_{\ell h} + \cdots$$  \hspace{1cm} (4)

where

$$J_\lambda = j_\lambda + j_\lambda^* + S_\lambda ,$$  \hspace{1cm} (5)
\[ j_\lambda = (j_\lambda)_\mu + (j_\lambda)_e \, , \] (6)

\( j_\lambda \), \( S_\lambda \) are, respectively, the strangeness conserving and the strangeness non-conserving parts of the hadron currents, and the \( \cdots \) denotes part of the non-leptonic interaction which, because of the observed \( |\Delta I| = \frac{1}{2} \) rule, does not appear to contain the complete \( \mathcal{L}_{hh} \).

All known weak reactions are consistent with the following selection rules:

<table>
<thead>
<tr>
<th>Accuracy</th>
<th>Selection Rule</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exact</td>
<td>( \Delta Q = \Delta N = \Delta L_e = \Delta L_\mu = 0 )</td>
</tr>
<tr>
<td>( \sim 10^{-6} )</td>
<td>(</td>
</tr>
<tr>
<td>( \sim \alpha )</td>
<td>(</td>
</tr>
<tr>
<td>( \sim 10^{-1} )</td>
<td>( \mathcal{L}_{gh} \sim SU_3 ) octet \n</td>
</tr>
</tbody>
</table>

The accuracy represents only the theoretical expectations. In many cases (e.g., the \( \Delta Q_h = \Delta S \) rule), the theoretical expectations often far exceed the actual experimental verifications.

In the above table, \( Q, N \) and \( S \) are, respectively, the total charge, the baryon number and the strangeness, \( Q_h \) is the total charge of the hadrons (in units of \( e \)), \( L_e \) and \( L_\mu \) are the two leptonic numbers defined by
\[ L_\lambda = \begin{cases} 
+1 & \text{for } L^- \text{ and } \bar{\nu}_e \\
-1 & \text{for } L^+ \text{ and } \bar{\nu}_e \\
0 & \text{otherwise} 
\end{cases} \quad (7) \]

For all \( \Delta S \neq 0 \) processes, the iso-spin \( I \) cannot be conserved. The \( I \Delta I \leq 1 \) rule, therefore, implies the usual \( I \Delta I = \frac{1}{2} \) rule for all \( \Delta S \neq 0 \) reactions; it also implies the usual \( I \Delta I = 1 \) rule for the \( \Delta S = 0 \) leptonic processes. The \( V_\lambda \) refers to the vector part of the strangeness conserving current \( J_\lambda \), and \( (J_\lambda^{el}) \) is proportional to the iso-vector part of the electromagnetic current of the hadrons. Under an iso-spin rotation \( V_\lambda \), \( (J_\lambda^{el}) \) and \( V_\lambda^* \) transform like the three members of an \( I = 1 \) triplet. From this iso-triplet property it follows that the vector current is conserved; i.e.

\[ \frac{\partial V_\lambda}{\partial x_\lambda} = 0. \quad (8) \]

In the limit of \( SU_3 \) symmetry, the currents \( J_\lambda \) and \( S_\lambda \) are assumed to be proportional to members of a single octet current operator \( (O_\lambda)^i_j \):

\[ J_\lambda \propto (O_\lambda)^1_2 \]

and

\[ S_\lambda \propto (O_\lambda)^1_3. \quad (9) \]

Furthermore, it is assumed that the same proportionality constant holds for both the vector and the axial-vector part of \( J_\lambda \) and \( S_\lambda \). Let us denote the proportionality constant by \( \tan \theta \); then, by definition,

\[ \tan \theta = \frac{g_S}{g_V} \quad (10) \]

where \( g_V \) is the Fermi constant in the usual \( \beta \)-decay (i.e. \( n \rightarrow p + e^- + \nu_e \)) and
$g_S$ is the corresponding vector coupling constant in the strangeness non-conserving decays. It is found that the angle $\theta$, called the Cabibbo angle, is given by

$$101 \equiv .24$$

if $\pi_{e3}$ and $K_{e3}$ decay rates are used, and

$$101 \equiv .26$$

if a least square fit is used to include all existing experiments.\(^5\)

1. 2. Coupling Constants and Renormalization

A. Radiative Corrections of $(g_V/g_\mu)$

It has been suggested that\(^3,6\)

$$g_V^2 + g_S^2 = g_\mu^2 .$$  \hspace{1cm} (11)

Thus, the Cabibbo angle $\theta$ is also given by

$$\cos \theta = \frac{g_V}{g_\mu} .$$  \hspace{1cm} (12)

From Eq. (10), the value of $\theta$ has already been determined. By using Eq. (12) and $\theta$, the ratio $(g_V/g_\mu)$ can be calculated. One finds

$$\left( \frac{g_\mu - g_V}{g_\mu} \right)_{\text{th}} \approx \begin{cases} 2.9\% & \text{if } 101 \equiv .24 \\ 3.4\% & \text{if } 101 \equiv .26 \end{cases}$$  \hspace{1cm} (13)

Now, without radiative correction, the observed value is\(^7\)

$$\left[ \frac{g_\mu - g_V}{g_\mu} \right]_{\text{uncor.}} = 1.19\% .$$  \hspace{1cm} (14)

Since the vector current is conserved, there is no correction due to the strong interaction. The electromagnetic interaction, however, does introduce a correction term.
We may write

\[
\left[ \frac{g_\mu - g_V}{g_\mu} \right]_{\text{cor}} = 1.19\% + r \tag{15}
\]

where \( r \) is the radiative correction. The radiative correction for the \( \mu \)-decay is straightforward; that of the \( \beta \)-decay is complicated because of the strong interaction. In the following, we give only the result for the "bare" nucleon.

In the Fermi theory, \( r \) is divergent: 8

\[
(r)_{\text{Fermi}} = \frac{\alpha}{4\pi} \left[ \pi^2 - \frac{25}{4} + 6 \ln \left( \frac{\lambda}{m_p} \right) + 3 \ln \left( \frac{m_p}{2E_m} \right) - 2.85 \right] \tag{16}
\]

where \( m_p \) is the mass of the proton, \( E_m \) is the maximum \( \beta \) energy and \( \lambda \) is the high energy cut-off parameter. The presence of the strong interaction might provide such a cut-off.

If we assume the existence of an intermediate boson \( W^\pm \), then \( r \) becomes finite, and it is given by 9

\[
(r)_W = \frac{\alpha}{4\pi} \left[ \pi^2 - \frac{25}{4} + 6 \ln \left( \frac{m_W}{m_p} \right) + 3 \ln \left( \frac{m_p}{2E_m} \right) - 2.85 \right] - \frac{3}{10} \left( \frac{m_\mu}{m_W} \right)^2 \tag{17}
\]

where \( m_W \) is the mass of \( W^\pm \). This finite result can be obtained by using only the lowest order perturbation method (without using the \( \xi \)-limiting process). Eq. (17) is, again, derived only for the bare nucleon. [If the cut-off parameter \( \lambda \) due to the strong interaction is \( > m_W \), then Eq. (17) may be applicable to a physical nucleon.]

From the high energy neutrino experiments, we know that

\[ m_W > 2 \text{ Gev} \, , \]
and, therefore,

\[
\left[ \frac{g_\mu - g_V}{g_\mu} \right]_{\text{cor}} > 2.4\% \quad (18)
\]

which is consistent with Cabibbo's result, Eq. (13).

We may also assume that Cabibbo's result is correct and use Eq. (13) and Eq. (17) [or Eq. (16)] to obtain an estimation of \( m_W \) [or \( \lambda \)]. We find

\[
m_W \quad [\text{or} \quad \lambda] \approx \begin{cases} 
8 \ m_p & \text{if } |\theta| \approx .24 \\
30 \ m_p & \text{if } |\theta| \approx .26 
\end{cases} \quad (19)
\]

In the latter case, and if \( W^\pm \) exists, the dimensionless weak interaction coupling constant becomes comparable to that of the electromagnetic interaction. Of course, Eq. (19) can only be regarded as, at best, indicative.

The significance of the equality \( g_V^2 + g_S^2 = g_\mu^2 \) can be best illustrated\(^9\) by using a quark model\(^{10}\) or any one of the triplet models for the hadrons. Let \( \psi_1, \psi_2, \psi_3 \) be the field operators for the three quarks. The current \( \bar{q}_\lambda \) becomes simply

\[
\bar{q}_\lambda = \psi_2^* \gamma_4 \gamma_\lambda (1 + \gamma_5) \psi_1 + \psi_e^* \gamma_4 \gamma_\lambda (1 + \gamma_5) \psi_e + \psi_\mu^* \gamma_4 \gamma_\lambda (1 + \gamma_5) \psi_\mu \quad (20)
\]

where \( \psi_2 = \cos \theta \psi_2 + \sin \theta \psi_3 \). Eq. (20) shows clearly the symmetry between the leptons and the hadrons. However, the extension of such symmetry to the non-leptonic weak interaction is still unclear.

B. Renormalization Constant of \( (g_A/g_V) \)

Recently, Adler and Weissberger\(^{11}\) have obtained an approximate expression for the renormalized \( (g_A/g_V) \) in \( \beta \)-decay. They started from the assumption that the
axial vector current $A_\lambda$ in $\beta$-decay satisfies the commutation relation\(^\text{12}\)

\[
[ \int A_4^* \, d^3 r , \int A_4 \, d^3 r ] = 2 I_z
\]

(21)

where $I_z$ is the $z$-component of the iso-spin operator. Such commutation rule follows, e.g., if we use the expression of $\frac{\delta A_\mu}{\delta x_\mu}$ given by Eq. (20). By taking into account only the part of $\frac{\delta A_\mu}{\delta x_\mu}$ that is proportional to the single pion field operator\(^\text{13}\) $\phi_\pi$, Adler and Weissberger derived the following expression for $\frac{g_A}{g_V}$:

\[
\left| \frac{g_A}{g_V} \right| = \left( 1 + \frac{4 m_N^2}{m_N} \int \frac{W \, dW}{W^2 - m_N^2} \left[ \sigma_{\text{tot}}^{-}(W) - \sigma_{\text{tot}}^{+}(W) \right] \right)^{-\frac{1}{2}}
\]

(22)

where $(f^2_{\pi N}/4\pi) \approx 14.6$ is the pion-nucleon coupling constant, $m_N$ is the mass of the nucleon and $\sigma_{\text{tot}}^{\pm}(W)$ is the total cross-section for scattering of a zero-mass $\pi^\pm$ on a proton at a center-of-mass energy $W$. The evaluation of Eq. (21) involves some off-mass-shell extrapolations of the observed scattering cross-sections. Their results are

\[
\left| \frac{g_A}{g_V} \right| \approx \begin{cases} 
1.24 & \text{Adler} \\
1.16 & \text{Weissberger}
\end{cases}
\]

(23)

which agree quite well with the observed value $\left( \frac{g_A}{g_V} \right)_{\text{exp}} = -1.18 \pm .02$.

Similar calculations have been extended by Amati, Bouchiat and Nuyts\(^\text{14}\) and by Levinson and Muzininich\(^\text{15}\) to include the leptonic decays of the hyperons.

1.3. Symmetry Properties of the Leptons

As is well known, except for the mass difference between $\mu$ and $e$, all known interactions, electromagnetic and weak, are symmetric with respect to the exchange

$\mu \leftrightarrow e$


\[ \nu_\mu = \nu_e . \]  

(24)

This suggests that it may be meaningful to regard the masses \( m_\mu \) and \( m_e \) as not due to the weak interaction [and, perhaps, also not due to the electromagnetic interaction].

In this section, we will study the symmetry properties of \( H_{\text{free}} + H_{\text{weak}} \) in the limit

\[ m_\mu = m_e = 0 . \]  

(25)

We may decompose the lepton field operators

\[ \psi_\ell = \ell_L + \ell_R \]  

(26)

where

\[ \ell_L = \frac{1}{2} \left( 1 + \gamma_5 \right) \psi_\ell , \]

\[ \ell_R = \frac{1}{2} \left( 1 - \gamma_5 \right) \psi_\ell , \]  

(27)

and \( \ell = e \) and \( \mu \).

We note that the leptonic weak interaction current \( j_\lambda \) and the leptonic electromagnetic current \( j_\lambda^{\text{el}} \) are both invariant under a \( U_2 \times U_2 \) group: 16, 17

\[
\begin{pmatrix} e_L \\ \mu_L \end{pmatrix} \rightarrow U \begin{pmatrix} e_L \\ \mu_L \end{pmatrix},
\]

\[
\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} \rightarrow U \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix},
\]

\[
\begin{pmatrix} e_R \\ \mu_R \end{pmatrix} \rightarrow U \begin{pmatrix} e_R \\ \mu_R \end{pmatrix},
\]

\[
\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} \rightarrow U \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}. \]  

(28)
where $u$ and $v$ are two arbitrary $(2 \times 2)$ unitary matrices.

Furthermore, there exists a discrete transformation $D$ under which

$$D \ \Phi_{\mu} D^{-1} = \Phi^*_{\mu}. \quad (29)$$

The $D$-transformation exchanges

$$\nu_e = e_L$$

$$\nu_\mu = \mu_L$$

and

$$\psi_1 = \psi_2 \quad (30)$$

where $\psi_1$ and $\psi_2$ are the quark fields used in Eq. (20). The transformation properties of the known hadrons under $D$ can be readily obtained by using Eq. (30). [The existence of $D$-symmetry, of course, does not depend on the quark model.] Thus, the leptonic part of the weak interaction is symmetric under $D$ and the $U_2 \times U_2$ group of transformations.

The $D$-symmetry can be easily extended to the non-leptonic part of the weak interaction as well. The weak interaction is, then, invariant under a group $G_{\text{wk}}$ of transformations where $G_{\text{wk}}$ is the extension of the $U_2 \times U_2$ group by using the discrete element $D$.

The $U_2 \times U_2$ group of symmetry transformations is violated by the masses $m_\mu$ and $m_e$, and the $D$-symmetry is violated by the masses of the leptons, by the electromagnetic interaction and by the $SU_3$ violating part of the strong interactions. However, both symmetries could be valid to all orders of the weak interaction.

The operator $D$ changes the weak interaction current $\Phi_{\mu}$ to its (hermitian) conjugate current $\Phi_{\mu}^*$; therefore, the operator $D$ has a rule in the weak interaction which is quite similar to that of the charge conjugation operator in the electromagnetic interaction.
II. Questions of $C, P, T$ Non-invariances

II.1. Two $\pi$ Decay Mode of $K_2^0$

As you all know, it was discovered last year by Christenson, Cronin, Fitch
and Turlay that the long-lived component $K_2^0$ of the neutral $K$ meson has a $2\pi$
decay mode

$$K_2^0 \rightarrow \pi^+ + \pi^-.$$  \hspace{1cm} (31)

Since both the particle anti-particle conjugation operator $C$ and the space inversion $P$
in the center of mass system interchange $\pi^+$ and $\pi^-$, the final $(2\pi)$ system in this
decay must be of $CP = +1$. On the other hand, the same long-lived component $K_2^0$
also decays into $3\pi$,

$$K_2^0 \rightarrow \pi^+ + \pi^- + \pi^0$$  \hspace{1cm} (32)

in which the $\pi^+$ and $\pi^-$ are observed to be predominantly in the S state. Thus, the
final $3\pi$ state is (or, at least, predominantly is) of $CP = -1$.

Now, $K_2^0$ is, by definition, one with a definite lifetime. The fact that it can
decay into different states with opposite CP values shows conclusively that CP is not
conserved in the $K_2^0$ decays. The CP violation is, therefore, established independently
of the detailed theory of the $K_1^0, K_2^0$ system.

The physical $K_2^0$ state and its decay processes are determined by the total
Hamiltonian $H$. The observed CP non-invariance implies that

$$[H, CP] \neq 0$$  \hspace{1cm} (33)

where

$$H = H_{st} + H_{y} + H_{wk}.$$  \hspace{1cm} (34)

From this single experiment it is, of course, not possible to conclude which part of $H$
is responsible for this CP violation; whether it is due to the strong interaction \(^{20}H_{\text{st}}\), or the electromagnetic interaction \(^{21,22}H_{\gamma}\), or the weak interaction \(^{23}H_{\text{wk}}\) or a combination of different interactions. There is also the possibility of a sub-weak interaction, \(^{23}\) which however will not be discussed here.

The amplitude of this new decay mode is very small:

\[
\epsilon = \left| \frac{\text{Rate} \left( K_2^0 \rightarrow \pi^+ + \pi^- \right)}{\text{Rate} \left( K_1^0 \rightarrow \pi^+ + \pi^- \right)} \right|^{1/2} \approx 2.2 \times 10^{-3} \tag{35}
\]

where \(K_1^0\) refers to the short-lived component of \(K\) meson. The smallness of the violation parameter \(\epsilon\) is one of the striking features of this new discovery.

II. 2. Experimental Foundation of C, P, T Invariances

It is well known that parity is not conserved in the weak interaction. Since the discovery of the non-conservation of parity in weak interactions, there have been many experiments to search for possible P violating effects in nuclear reactions. \(^{24}\) These experiments establish that the magnitudes of the P non-conserving amplitudes are smaller than that of the corresponding P conserving amplitudes by, at least, a factor \(\sim 10^{-5}\), which is about the order of magnitude of the dimensionless weak coupling constant \(g_v m_p^2\). Thus, we should regard both strong and electromagnetic interactions to be P conserving.

Invariance under CPT implies \(^{25}\) mass and lifetime equalities between any particle and its anti-particle. In view of the possible large violations of C and CP invariances we may use such equalities as evidence for CPT invariance. Among such equalities, the most accurate one is that between \(K^0\) and \(\bar{K}^0\):
\[ < K^0 | H_{st} + H_\gamma + H_{wk} | K^0 > = < \overline{K}^0 | H_{st} + H_\gamma + H_{wk} | \overline{K}^0 > \]  

From the experimental mass difference \( \Delta m \) between \( K_1^0 \) and \( K_2^0 \), we conclude that Eq. (36) holds to \( \sim 10^{-14} \). Thus, we should regard \( H_{st} \) and \( H_\gamma \) to be both invariant under CPT to an accuracy \( \sim 10^{-14} \) and \( \sim 10^{-12} \), respectively.

Eq. (36) also shows that the \( \Delta S = 0 \) non-leptonic part of \( H_{wk} \) conserves CPT to an accuracy \( \sim 10^{-8} \). For the other parts of \( H_{wk} \), evidence for the CPT invariance is given by the upper limits on the lifetime difference \( \Delta \tau \) between that of a particle and its anti-particle: \( \Delta \tau \) \( \sim 10^{-2} \) in relative amplitudes the strong interaction is invariant under the particle anti-particle conjugation \( C \); i.e.

\[ [ H_{st}, C ] = 0. \]

Among these experimental evidences, we may mention the recent study by the Columbia group on the equality between the energy distributions of \( \pi^+ \) and \( \pi^- \) in the annihilation of \( \bar{p} \) and \( p \),

\[ \bar{p} + p \rightarrow \pi^+ + \pi^- + \ldots \]  

which places an upper limit on the \( C \) non-invariant amplitude to be not more than \( \sim 1\% \) of the \( C \) invariant amplitude. A similar upper limit of \( \sim 2\% \) is obtained by studying the energy distributions of \( K^+ \) and \( K^- \) in the same \( (\bar{p} + p) \) annihilation experiment. Further evidence of \( C \) invariance of the strong interaction comes from the smallness of the CP violation parameter \( \epsilon \) in \( K_2^0 \) decay. Additional supporting
evidence can also be obtained from the \( p-p \) scattering experiments\(^{29}\) and from the experiments on reciprocity relations in nuclear reactions.\(^{30}\)

There exists also some evidence that, within certain experimental accuracies, the weak interaction is also invariant under \( CP \) or \( T \):

For the \( \Delta S \neq 0 \) non-leptonic weak interaction we can use the smallness of \( \epsilon \) to conclude that within an accuracy of \( \sim 10^{-2} \) it is invariant under \( CP \); otherwise, the observed amplitude of \( K^0 \rightarrow \pi^+ + \pi^- \) would be much larger.

For the \( \Delta S \neq 0 \) leptonic weak interaction, the experimental limits on \( CP \) invariance are, at present, unclear. The detailed experimental status was summarized by Dr. Steinberenger this morning.

For the \( \Delta S = 0 \), leptonic weak reactions, the relative phase \( \phi \) between the \( \beta \)-decay coupling constants \( g_A \) and \( g_V \) has been measured, and is found to be\(^{31}\)

\[
\phi = 180^\circ \pm 8^\circ
\]  

(39)

which is consistent with \( T \) invariance.\(^{32}\) If "\( CPT \)" invariance is assumed, to the same accuracy we also have \( CP \) invariance.

In contrast, there exists at present no evidence\(^{21,33}\) that \( H_Q \) of the strongly interacting particles is, or is not, invariant under \( C \) or \( T \).

(a) Nucleon Form Factor

The most extensively studied problem concerning the electromagnetic property of the strongly interacting particles is the nucleon form factor. From proper Lorentz invariance and space inversion, the matrix element of the electromagnetic current \( \mathbf{j}_\mu^{el} \) is given by

\[
<N' | \mathbf{j}_\mu^{el} | N> = i U_{N'}^\dagger \gamma_\mu \left[ \gamma_\mu F_1 \sigma_{\mu\nu} q_\nu F_2 + q_\mu F_3 \right] U_N.
\]  

(40)
where $q_\mu$ is the 4-momentum transfer. From Hermiticity, we have $F_1$, $F_2$, $F_3$ to be all real. From T invariance, $F_1$, $F_2$ are real but $F_3$ is imaginary. Hence, $F_3 = 0$ if T invariance holds. However, it follows from

$$\frac{\partial q_\mu}{\partial x_\mu} = 0 \quad ,$$

(41)

$F_3 = 0$ already. Thus, no information on T invariance can be obtained from $e^- + p \rightarrow e^- + p$ experiment.

(b) Nuclear $\gamma$ Transition

The nuclear electromagnetic property is determined by that of its constituent nucleons and the strong nuclear forces. Independent of whether $H_\gamma$ is, or is not, invariant under T, there should be no T violating effect in nuclear $\gamma$ transitions within an accuracy $\sim1\%$, provided that $H_{st}$ is invariant under T.

An extensive study made in collaboration with Bernstein and Feinberg shows that at present there exists no direct or indirect evidence that $H_\gamma$ of the strongly interacting particles is or is not invariant under C.

The following table summarizes these results:

<table>
<thead>
<tr>
<th></th>
<th>$H_{st}$</th>
<th>$H_\gamma$</th>
<th>$H_{wk}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CPT</td>
<td>$\checkmark$</td>
<td>$\checkmark$</td>
<td>$\checkmark$</td>
</tr>
<tr>
<td>$P$</td>
<td>$\checkmark$</td>
<td>$\checkmark$</td>
<td>$X$</td>
</tr>
<tr>
<td>$C$</td>
<td>$\checkmark$</td>
<td>$?$</td>
<td>$X$</td>
</tr>
<tr>
<td>T or CP</td>
<td>$\checkmark$</td>
<td>$?$</td>
<td>$\checkmark$ (?)</td>
</tr>
</tbody>
</table>

The conclusion is quite clear: to explain the $K^0_L \rightarrow 2\pi$ we may invoke a small
CP non-invariant amplitude in $H_{st}$, or in $H_{wk}$ (if the observed parameter $\epsilon$ is a typical example of the CP violation). There is no difficulty in doing this, but one is only puzzled by the presence of such a small violation.

An alternative, and, perhaps, much more attractive, possibility is to assume that $H_{st}$ and $H_{wk}$ are invariant under CP, but $H_{\gamma}$ has a large violation of C invariance. In this case, all strong and weak processes can have a small C and CP non-invariant amplitude through virtual emission and absorption of $\gamma$. In particular, $K_{2}^0$ can decay into $2\pi$ with a fractional amplitude

$$\epsilon \approx \frac{\alpha}{\pi}.$$  \hspace{1cm} (42)

On the other hand, it has been well established that, at least for the leptons, the electromagnetic interaction $H_{\gamma}$ is invariant under the charge conjugation operator, which shall be denoted by $C_{\gamma}$. The possible non-invariance of $H_{\gamma}$ under the particle anti-particle conjugation operation $C$, naturally, leads one to inquire whether $C$ is, or is not, the same as $C_{\gamma}$.

II. 3. A Unifying View of Discrete Symmetry Violations

We note that if the identity of a particle could be taken for granted, then it would be possible to define $T$, the pure time reversal, and $P$, the pure space inversion, unambiguously. However, the distinguishability between different particles depends on their interactions, and degeneracies occur if certain interactions are absent. It is, therefore, not possible to give a unique definition of $P$ and $T$ without any reference to some specific interactions.
To illustrate this point, let us consider the following two examples of weak decays.

(i)
\[ K^+ \rightarrow \begin{cases} \pi^+ + \pi^0 \\ \pi^+ + \pi^+ + \pi^- \end{cases} \]  
(43)

This is the well known \( \theta - \tau \) puzzle, based on which the original theoretical proposal of the non-conservation of parity was first suggested. However, if we could switch off the strong and the electromagnetic interactions, then from reaction (43) alone, it would not be possible to conclude any parity non-conservation. If \( H_{st} = H_{\gamma} = 0 \), we could conclude from reaction (43) that the parity conservation holds and that the parities for the pions in \( S \)-states satisfy

\[ P(\pi^+ \pi^0) = P(\pi^+ \pi^+ \pi^-) \]  
(44)

Now, if \( H_{st} \) and \( H_{\gamma} \) are turned on, it would, then, be discovered that Eq. (44) is violated. Thus, inasmuch as we regard \( H_{wk} \) as violating the conservation of parity where parity \( (= P_{st} = P_{\gamma}) \) is determined by \( H_{st} \) and \( H_{\gamma} \), we could obtain the same result by regarding \( H_{wk} \) to be parity conserving, and \( H_{st} \) and \( H_{\gamma} \) to be parity violating where parity \( (= P_{wk}) \) is determined by the weak interaction.

(ii) Another example of parity non-conservation is the \( \pi_\mu \) decays:

\[ \pi^+ \rightarrow \mu^+ + \nu_\mu (L) \]
\[ \pi^- \rightarrow \mu^- + \bar{\nu}_\mu (R) \]  
(45)

where \( L \) and \( R \) denote the helicities.

In the absence of the electromagnetic interaction we might regard the physical pions as

\[ \pi_1 \equiv \left( \frac{1}{\sqrt{2}} \right) (\pi^+ + \pi^-) \]  
(46)
and

\[ \pi_2 \equiv \frac{1}{\sqrt{2}} (\pi^+ - \pi^-). \]  

(47)

The particle \( \pi_1 \), or \( \pi_2 \), decays with equal probability to the left-handed \( \nu_\mu \) and to the right-handed \( \bar{\nu}_\mu \), without any apparent violation of parity. Again, by considering only this weak reaction it is not possible to conclude that \( H_{wk} \) is not parity conserving.

Indeed, all known weak reactions are consistent with the assumption that \( H_{wk} \) is invariant under \( P_{wk} \), \( T_{wk} \), and \( C_{wk} \). A simple choice\(^{34}\) would be to define \( C_{wk} \) to be the operator \( D \), given by Eq. (29) which conjugates the weak interaction current \( \tilde{\psi}_\mu \), \( T_{wk} = T_{st} \), and \( P_{wk} = D^{-1} C_{st} P_{st} \) where \( C_{st} \), \( T_{st} \), and \( P_{st} \) are determined by the strong interaction.

We now make the fundamental assumption: Each interaction\(^{35}\) \( H_i \) \((i = \text{strong}, \gamma \) and weak\) is separately invariant under its own \( P_i \), \( T_i \), and \( C_i \). This assumption is consistent with all the existing experiments. From the experimental evidences discussed in section II.2, we conclude that

\[ C_{st} T_{st} P_{st} = C_{st} T_{st} P_{st} = C_{wk} T_{wk} P_{wk} \]  

(48)

and

\[ P_{st} = P_{\gamma} \neq P_{wk}. \]  

(49)

Since reaction (38) is a strong process, the particle anti-particle conjugation operator \( C \) must be the same as \( C_{st} \); i.e.

\[ C_{st} = C; \]

furthermore, under \( C_{st} \)

\[ C_{st} |p> = |\bar{p}> , \]

\[ C_{st} |n> = |\bar{n}> , \]  

(50)
and, consequently,
\[ C_{st} \, \lambda^+ > = \lambda \pi^- > \, , \text{ etc.} \]

[The \( C_{st} \) may, for example, be regarded as the baryon number conjugation, and can, in principle, be different from the charge conjugation \( C_\gamma \).]

The possible \( C \) non-invariance of \( H_\gamma \) means simply the possible mis-match between the particle anti-particle conjugation \( C_{st} \) and the charge conjugation \( C_\gamma \); i.e.
\[ C_{st} \neq C_\gamma \, . \quad (51) \]

In this case, we may set
\[ T_{wk} = T_{st} \neq T_\gamma \, . \quad (52) \]

and, consequently, \( C_{wk} P_{wk} = C_{st} P_{st} \neq C_\gamma P_\gamma \). The decay \( K_2^0 \rightarrow \pi^+ \pi^- \) can be attributed to the \( C_{st} \) non-conservation of \( H_\gamma \), or the \( C_\gamma \) non-conservation of \( H_{st} \).

The following table illustrates these possibilities:

<table>
<thead>
<tr>
<th>( H_{st} )</th>
<th>( C_{st} )</th>
<th>( T_{st} )</th>
<th>( P_{st} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( H_\gamma )</td>
<td>( C_\gamma )</td>
<td>( T_\gamma )</td>
<td>( P_\gamma )</td>
</tr>
<tr>
<td>( H_{wk} )</td>
<td>( C_{wk} )</td>
<td>( T_{wk} )</td>
<td>( P_{wk} )</td>
</tr>
</tbody>
</table>

The mis-matches between the various operators, \( C_{st} \neq C_\gamma \neq C_{wk} \neq C_{st} \), \( P_{st} = P_\gamma \neq P_{wk} \) and \( T_{wk} = T_{st} \neq T_\gamma \), give rise to all the non-conservations of \( C \), \( P \) and \( T \).
Remarks.

1. We emphasize that the existing experimental evidences require that \( H_{st} \) be invariant, within an accuracy \( \sim 10^{-2} \), under \( C_{st} \), \( T_{st} \) and \( P_{st} \); the presently used form of \( H_{wk} \) is invariant under \( C_{wk} \), \( T_{wk} \) and \( P_{wk} \); if we assume that \( H_{\gamma} \) consists of only the minimal electromagnetic interaction of the spin 0 and \( \frac{1}{2} \) particles, then \( H_{\gamma} \) is automatically invariant under \( C_{\gamma} \), \( T_{\gamma} \) and \( P_{\gamma} \).

2. It is interesting to note that our usual \( T \) and \( P \) often refer to \( T_{\gamma} \) and \( P_{\gamma} \). This is because at large distances, due to its long-range character, the electromagnetic force predominates. Thus, in all collision processes, the asymptotic conditions are determined by the physical masses and the electromagnetic properties of the incoming and outgoing particles. It is, therefore, convenient to identify \( T = T_{\gamma} \) and \( P = P_{\gamma} \), since \((H_{\text{free}} + H_{\gamma})\) is invariant under both \( T_{\gamma} \) and \( P_{\gamma} \). On the other hand, the internal structures of the non-leptons are determined mainly by \( H_{st} \), which leads us to identify the particle anti-particle conjugation operator \( C \) with \( C_{st} \). However, such identifications may lead to unnecessary confusion if \( C_{st} \neq C_{\gamma} \). The \( H_{st} \) and \( H_{\gamma} \) are, then, not invariant under \( C_{st} T_{\gamma} P_{\gamma} \), which would be identified as CTP; yet, the "CTP" Theorem remains satisfied, since all these interactions are invariant under \( C_{st} T_{st} P_{st} = C_{\gamma} T_{\gamma} P_{\gamma} = C_{wk} T_{wk} P_{wk} \).

3. The leptons have no strong interaction; their electromagnetic interaction is, therefore, invariant under \( C = C_{\gamma} \), \( P = P_{\gamma} \) and \( T = T_{\gamma} \) separately.

4. As remarked before, the definition of the discrete space-time transformation "P" and "T" cannot be given without any reference to a specific interaction. Such ambiguity, however, does not exist for the continuous inhomogeneous Lorentz transformations. This may be regarded as one of the reasons why the invariance under the continuous space-time transformations holds better than the discrete space-time transformations.
II. 4. Classification of $C_{st}$: Non-invariant Electromagnetic Interaction

Let $\mathcal{J}^{	ext{el}}$ be the total electromagnetic current which can be decomposed to a lepton part $(\mathcal{J}^{	ext{el}})^L$ and a non-leptonic (i.e., hadronic) part $(\mathcal{J}^{	ext{el}})^h$:

$$(\mathcal{J}^{	ext{el}})^L = (\mathcal{J}^{	ext{el}})^L + (\mathcal{J}^{	ext{el}})^h.$$  \hfill (53)

The operator $C_{st}$ applies only to the hadrons. Let us define

$$2I_\lambda = (\mathcal{J}^{	ext{el}})^h - C_{st}(\mathcal{J}^{	ext{el}})^L C_{st}^{-1}$$  \hfill (54)

and

$$2K_\lambda = (\mathcal{J}^{	ext{el}})^h + C_{st}(\mathcal{J}^{	ext{el}})^L C_{st}^{-1}.$$  \hfill (55)

Thus, we can write

$$(\mathcal{J}^{	ext{el}})^h = I_\lambda + K_\lambda$$  \hfill (56)

where

$$C_{st} I_\lambda C_{st}^{-1} = -I_\lambda$$  \hfill (57)

and

$$C_{st} K_\lambda C_{st}^{-1} = +K_\lambda.$$  \hfill (58)

By definition, all electromagnetic currents change sign under $C_\gamma$; i.e.

$$C_\gamma (\mathcal{J}^{	ext{el}})_{\alpha} C^{-1}_\gamma = -(\mathcal{J}^{	ext{el}})_{\alpha}$$  \hfill (59)

where $\alpha = L$ or $h$. A mis-match between $C_{st}$ and $C_\gamma$ means that

$$K_\mu \neq 0$$  \hfill (60)

and vice versa.

We note that, since $H_{st}$ conserves both $(\mathcal{J}^\mu)^h$ and $C_{st}$, it must also conserve $K_\mu$. Thus, the electric charge associated with $K_\mu$ is conserved by $H_{st}$; i.e.

$$[ Q_{K_\mu}, H_{st} ] = 0.$$  \hfill (61)
where

\[ Q_K = -i \int K_4 d^3 r. \quad (62) \]

All possible \( C_{st} \) non-invariant \( H_y \) can, thus, be classified, according to whether the operator \( Q_K \) is zero or not, into the following two classes:

(A) \( Q_K = 0 \) identically.

As an example, we may choose

\[ K_\mu = i \lambda \frac{2}{gb_{\nu \nu}} (\bar{\phi}_\mu \omega_\nu - \bar{\phi}_\nu \omega_\mu) \quad (63) \]

where \( \lambda \) is a real constant and \( \phi_\mu \), \( \omega_\mu \) are the field operators for the spin 1 particles \( \phi^0 \) and \( \omega^0 \) respectively. Such a \( K_\lambda \) is connected with the mixed magnetic moment current, and it gives rise to the decay

\[ \phi^0 \rightarrow \omega^0 + \gamma. \quad (64) \]

Its 4th component is the divergence of a 3-vector which leads to \( Q_K = 0 \).

From the strong reactions such as \( \phi^0 \rightarrow K_1^0 + K_2^0 \) and \( \omega^0 \rightarrow \pi^+ + \pi^- + \pi^0 \), we determine that

\[ C_{st} (\phi^0) = C_{st} (\omega^0) = -1. \quad (65) \]

Consequently,

\[ C_{st} K_\mu C_{st}^{-1} = + K_\mu. \quad (66) \]

However, from Eq. (64) and the requirement that \( H_y \) is invariant under \( C_y \), we find

\[ C_y (\phi^0) = -C_y (\omega^0) \quad (67) \]

and

\[ C_y K_\mu C_y^{-1} = -K_\mu. \quad (68) \]

In this example, \( K_\mu \) transforms like a unitary octet under the \( SU_3 \) group of transformations.
(8) \( Q_K \neq 0 \).

The simplest example of this class of electromagnetic interactions is to assume the existence of a new strongly interacting charged, but \( C_{st} = 1 \), particle \( a^\pm \) which is of \( I = N = S = 0 \). We assume that the strong interaction between \( a^\pm \) and the known particles is of such a form that

\[
C_{st} \left| a^+ \right> = \left( a^+ \right> \tag{69}
\]

and

\[
T_{st} \left| a_{k, \frac{1}{2}}^+ \right> = \eta \left| a_{-k, -\frac{1}{2}}^- \right> \tag{70}
\]

where \( \eta \) is a phase factor. The existence of the \( a^\pm \) particle with such a strong interaction, then, necessitates the mis-match between \( C_{st} \) and \( C_Y \) and between \( T_{st} \) and \( T_Y \).

In this case, the current \( K_\mu \) transforms like a singlet under either the \( SU_3 \) or the iso-spin transformations. [In footnote 36, it is proved that under very general conditions, the current \( K_\mu \) of these \( a^\pm \)-particles remains a unitary singlet, even if the \( a \)'s are not themselves unitary singlets.]

Among the various possible tests of the possible \( C_{st} \) and \( T_{st} \) non-invariances of \( H_Y \), we list a few examples:

(i) There can be asymmetries between the energy distributions of \( \pi^+ \) and \( \pi^- \) in,

\[
\eta^0 \rightarrow \pi^+ + \pi^- + \gamma \tag{71}
\]

Other decays such as \( \eta^0 \rightarrow \pi^+ + \pi^- + \pi^0 \) and the decays of \( \eta' \) can also be used\(^{37} \) to test the \( C_{st} \) and \( T_{st} \) invariances of \( H_Y \).

(ii) There can be correlations such as

\[
J \cdot (k \times k') \quad \text{and} \quad J \cdot (k \times k') \tag{72}
\]

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in the single photon exchange approximation of the inelastic reaction

\[ e^- + N \rightarrow e^- + N^* \]  

(73)

where \( k, k' \) are, respectively, the initial and the final momenta of \( e^- \), and \( N^* \) are, respectively, the spin vectors of \( N \) and \( N^* \).

(iii) \( \ln^3 \pi^- + p \rightarrow n + e^+ + e^- \)

(74)

there can be correlation, in the single photon exchange approximation, between the spin of the neutron and the vector \( k_+ \times k_- \) where \( k_\pm \) are the momentum of \( e^\pm \).

(iv) If the particle \( a^\pm \) exists, then it can be produced strongly; e.g.,

\[ \bar{p} + p \rightarrow a^+ + a^- + \pi^+ + \pi^- + \cdots \]  

(75)

Under \( C_s \), \( p \rightarrow \bar{p}, \bar{p} \rightarrow p, \pi^\pm \rightarrow \pi^\mp \) but \( a^+ \rightarrow a^+ \) and \( a^- \rightarrow a^- \). Thus, there can be large asymmetry between the energy distributions of \( a^+ \) and that of \( a^- \); i.e.

\[ dN \left( E_{a^+} = E_1, \quad E_{a^-} = E_2 \right) \neq dN \left( E_{a^+} = E_2, \quad E_{a^-} = E_1 \right). \]  

(76)

Quite likely, the \( a^\pm \), if it exists, may decay through the weak interaction. If we assume that its weak interaction satisfies both the D-symmetry and the octet current hypothesis, then many of its branching ratios become uniquely determined. For example, if the \( a^\pm \) is a boson (denoted by \( a^\pm_B \)), we find, in the limit of \( SU_3 \) symmetry and assuming that \( a^+ \rightarrow a^- \) under \( D = C_{wk} \),

\[
\begin{align*}
\text{Rate } (a^+_B \rightarrow \eta^0 + \ell^+ + \nu_\ell) & \propto \frac{1}{6} (3 \cos^2 \theta - 1)^2 \\
\text{Rate } (a^+_B \rightarrow K^0 + \ell^+ + \nu_\ell) & \propto \sin^2 \theta \cos^2 \theta \\
\text{Rate } (a^+_B \rightarrow \bar{K}^0 + \ell^+ + \nu_\ell) & \propto \sin^2 \theta \cos^2 \theta \\
\text{and} \quad \text{Rate } (a^+_B \rightarrow \pi^0 + \ell^+ + \nu_\ell) & \propto \frac{1}{2} \sin^4 \theta
\end{align*}
\]

(77)
with the same common proportional constant. The angle $\theta$ is the Cabibbo angle.

Identical results can be derived if the $a^\pm$ is a Fermion (denoted by $a^\pm_F$) by, respectively, replacing $\eta^0, \kappa^0, R^0, \pi^0$ by $\Lambda^0, n, \Xi^0$, and $\Xi^0$ (or by $\bar{\Lambda}^0, \bar{n}, \bar{\Xi}^0, \bar{\Xi}^0$). The fact that we have not, as yet, observed the $a^\pm$ leads to a lower limit of 1.5 Gev for its mass $m_a$, depending on whether it is a Boson or a Fermion.

In conclusion, we emphasize that the possibility that $H_\gamma$ has large violation of $C_{st}$ is only a theoretical speculation, since it is based on just one experimental result: $e \equiv (\alpha/\pi)$. Nevertheless, the fact that there exists, at present, no evidence that $H_\gamma$ is, or is not, invariant under $C_{st}$ and $T_{st}$ should provide sufficient incentive for further experimental efforts in this direction.

Our concept of "C" has undergone several different evolutionary stages. It started with the charged conjugation operator $C_\gamma$ determined by the electromagnetic interaction of the electron. Later, the operator $C_\gamma$ was extended to the strongly interacting particles and became the particle anti-particle conjugation operator $C_{st}$. So long as everything is conserved, we may set these two conjugation operators equal. However, we now know that there must be non-conservations; it becomes, then, necessary to make the logical distinction between these two operators, and it is important to investigate which conjugation defined by what interaction is violated by what other interaction.
References

7. The $(g_\mu)_{\text{uncor.}}$ is related to the observed lifetime $\tau_\mu$ and the mass $m_\mu$ of the muon by $\tau_\mu^{-1} = (g_\mu)_{\text{uncor.}}^2 m_\mu^5 (192\pi^3)^{-1}$. From the values $m_\mu = (206.768 \pm 0.003) m_e$ and $\tau_\mu = (2.198 \pm 0.001) \times 10^{-6}$ sec. [Charpak et al., Phys. Letters 1, 16 (1962); Farley et al., Proc. of International Conference on High Energy Physics, CERN, Geneva (1962), p. 415], we find
\[ (g_\mu) = (1.4320 \pm 0.0011) \times 10^{-49} \text{ erg cm}^3. \]

The value of \( (g_\mu) \) can be determined from the various \( \beta \)-decay fit values, it was found that

\[ (g_\nu) = (1.4149 \pm 0.0022) \times 10^{-49} \text{ erg cm}^3. \]

[See C. S. Wu, Lecture Notes of International School of Physics "Enrico Fermi," Varenna, Italy (1964) for the detailed experimental references.]


27. $\mu^\pm$ lifetime:


$\pi^\pm$ lifetime:


$K^\pm$ lifetime:


32. It should be emphasized that, independent of $T$ invariance, $(g_A/g_V)$ must be real if the strangeness conserving current $J_{\mu}$ satisfies the charge symmetry condition $J_{\mu}^* = -\left[ \exp(i\pi I_{\gamma}) \right] J_{\mu} \left[ \exp(-i\pi I_{\gamma}) \right]$ where $I_{\gamma}$ is the $\gamma$-component of the iso-spin operator.
33. Recently, there have been several new experiments testing the $C_{st}$ invariance of $H_{\gamma}$: L. R. Price and F. S. Crawford, Phys. Rev. Letters 15, 123 (1965); J. S. Lindsay and G. A. Smith, Phys. Rev. Letters 15, 221 (1965); A. Rittenberg and G. R. Kalbfleisch, Phys. Rev. Letters 15, 556 (1965); C. Alff et al. (this conference).

Unfortunately, the conclusions that can be drawn from these experiments are still extremely limited. In particular, if the $C_{st} = +1$ current $K_{\mu}$ transforms either like a unitary octet or like a unitary singlet, or like an iso-scalar; then in the limit of $SU_3$ symmetry, or iso-spin rotation symmetry, $\eta^0 \not\to \pi^0 + e^+ + e^-$, in the single photon exchange approximation; similarly, there is no time reversal non-invariant effect in $\Sigma^0 \to \Lambda^0 + e^+ + e^-$, etc. [See the theoretical discussions by N. Cabibbo, Phys. Rev. Letters 14, 965 (1965); T. D. Lee, loc. cit.; G. Feinberg, Phys. Rev. (to be published).]

34. We note that this simple solution satisfies (i) $C_{wk} T_{wk} P_{wk} = C_{st} T_{st} P_{st}$, (ii) $C_{wk}^2 = 1$ and (iii) both $H_{st}$ and $H_{\gamma}$ violate $C_{wk}$ and $P_{wk}$ invariances. If we want to maintain these three conditions, then the general solution of $C_{wk}$, $T_{wk}$ and $P_{wk}$ can be obtained by multiplying this special form of $C_{wk}$, $T_{wk}$
and $P_{wk}$ by arbitrary members of the group $S_{wk}$, given in section 1.3, provided the conditions (i)-(iii) remain satisfied. [We may also try to find the general solution of $C_{wk}$, $T_{wk}$ and $P_{wk}$ without condition (iii); in this case, there exists a trivial solution: $C_{wk} = 1$, $P_{wk} = C_{st} P_{st}$ and $T_{wk} = T_{st}$.]

35. In some cases, the total interaction Hamiltonian $H$ cannot be expressed as a linear sum $(H_{st} + H_{\gamma} + H_{wk})$. For example, there may be derivative couplings in either $H_{st}$ or $H_{wk}$, which would generate an induced electromagnetic current with an amplitude proportional to $ef$ or $eg$ where $f$ and $g$ are, respectively, the appropriate strong and weak interaction coupling constants. The mathematical definitions of $H_{st}$, $H_{\gamma}$, $H_{wk}$ are, then, given by $H_{st} = \lim_{e=g=0} H_{st}$, $H_{\gamma} = \lim_{f=g=0} H_{\gamma}$ and $H_{wk} = \lim_{e=f=0} H_{wk}$.

36. To show this, let us consider first the functional relation between $Q_K$ and $I_z$, where $I_z$ is the $z$-component of the iso-spin operator. Since the eigen-values of both $Q_K$ and $I_z$ are additive quantum numbers, there can only be linear relations between these two operators; i.e., $Q_K = a I_z + b$. From $[Q_K, C_{st}] = [H_{st}, Q_K] = [H_{st}, C_{st}] = 0$, one can easily prove that all known particles have the eigen-value $Q_K = a I_z + b = 0$. Consequently, $a = 0$, and $Q_K$ is independent of $I_z$. By extending this argument to include other components of the iso-spin operator $I$, we establish that $[Q_K, I] = 0$. Similarly, it can be shown that $Q_K$ is a singlet under the $SU_3$ transformations.

In general, $Q_K$ being an $SU_3$ singlet, or an iso-scalar, does not mean that $K_\mu$ should transform in the same way. [For example, Eq. (63) gives an octet current $K_\mu$, but the $Q_K$ is zero, and, therefore, a singlet.] However, if we
assume that the electromagnetic current \( K_\mu \) is given only by the minimal electromagnetic interaction of the spin \( \frac{1}{2} \), or \( 0 \), particles, then \( K_\mu \) must also be a singlet under the SU\(_3\) transformations and the iso-spin transformations, and \( K_\mu \neq 0 \) implies that \( Q_K \neq 0 \).

It may be instructive to give an explicit example. Let us assume the existence of a unitary triplet of spin 0 or \( \frac{1}{2} \) particles \( (a_1^+, a_2^0, a_3^0) \) where all the \( a \)’s are of \( Q_K = +\frac{1}{2} \), the \( a_1 \) and \( a_2 \) are of \( I = \frac{1}{2} \), \( N = S = 0 \), and \( a_3 \) is of \( I = 0 \), \( N = 0 \) but \( S = -1 \). The total charge \( Q \) is given by \( Q = I_z + \frac{1}{2}(N + S) + Q_K \).

This triplet has, therefore, total charges \((+, 0, 0)\). It becomes one with total charges \((0, +, +)\) under \( C_{st} \) and a different set with total charges \((- , 0, 0)\) under \( T_{st} \). The \( C_{st} \) and \( T_{st} \) invariances, then, generate from the triplet \( (a_1^+, a_2^0, a_3^0) \) a total of four different sets of triplets. Nevertheless, the current \( K_\mu \), given by the minimal electromagnetic interaction of all these triplets, remains a unitary singlet.


38. This particular reaction was suggested to me by L. Lederman and M. Schwartz (private communication).

39. We note that reactions (77) satisfy the selection rule \( \Delta N = 0 \), \( \Delta Q_K = +1 \). The decay \( a_F^+ \rightarrow \Lambda^0 + \ell^+ + \nu_\ell \) satisfies the selection rule \( \Delta(N + Q_K) = 0 \), and the decay \( a_F^+ \rightarrow \bar{\Lambda}^0 + \ell^+ + \nu_\ell \) satisfies the selection rule \( \Delta(N - Q_K) = 0 \). Because

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of the stability of the known baryons, these selection rules cannot co-exist. Consequently, if both $a_F^\pm$ and $a_B^\pm$ exist, then one of them cannot have such decay modes.

40. If $a^\pm$ is stable, then a lower limit $m_a > 5$ Bev can be set by using the recent experimental results of D. E. Dorfan, J. Eades, L. M. Lederman, W. Lee and C. C. Ting, Phys. Rev. Letters 14, 999 (1965).
Space Inversion, Time Reversal and Particle-Antiparticle Conjugation

As we expand our observation, we extend our concepts. Thus the simple symmetries that once seemed self-evident are no longer taken for granted. Out of studies of different kinds of interactions we are learning that symmetry in nature is some complex mixture of changing plus into minus, running time backward and turning things inside out.

by T. D. Lee

The more we learn about symmetry operations — space inversion, time reversal and particle-antiparticle conjugation — the less we seem to understand them. At present, although still very little is known about the true nature of these discrete symmetries, we have, unfortunately, already reached the unhappy state of having lost most of our previous understanding. Let us, therefore, review the gradual evolution of our past concepts of these discrete symmetry operations.

P and T in classical physics

In classical mechanics, each particle is described by its space-time coordinates $r$ and $t$, and every particle is assumed to be different from every other particle. The space-inversion ($P$) and time-reversal ($T$) invariances in classical mechanics simply mean that the dynamical laws remain unchanged under

$$
P: \quad r \rightarrow -r, \quad t \rightarrow t$$

$$
T: \quad r \rightarrow r, \quad t \rightarrow -t
$$

(1)

Suppose we are given a record, say a movie record, of the motion of a system of particles. If $P$ invariance holds, then by examining only the movie it is not possible to decide with certainty whether the movie represents the true sequence or whether it represents the mirror image of the true sequence. Similarly, the $T$ invariance implies that if any movie of the motion of a system of particles is run backwards, then the time-reversed sequence also represents a possible solution of the dynamical equations.

For a macroscopic system with a large number of particles, although the time-reversed sequence is always a possible one if $T$ invariance holds, it is, in general, an improbable one. Thus although we cannot know for sure whether such a movie is being shown in its time-reversed order or not, we may try to guess. If the number of particles is sufficiently large, our guess will almost always be right. It is only in this statistical sense that we can differentiate for a macroscopic system any time-ordered sequence of events from its time-reversed sequence, and, thereby, determine the direction of our macroscopic time.

If the system contains only a very small number of particles, it is not possible, even in a statistical sense, to differentiate a time-ordered sequence from its time-reversed sequence (provided that $T$ invariance holds). As we shall see, this last statement has to be modified in quantum mechanics.

Nonconservation of parity

The symmetry operations of $P$ and $T$ in quantum mechanics were first studied and analysed by Eugene Wigner of Princeton. Both operations were
PARITY NONCONSERVATION. Cobalt-60 decays into nickel-60 plus an electron and an antineutrino. When the original nucleus is polarized, the emitted electron is found to have a left-handed spin, and its preferred direction of motion is opposite to the polarization direction of the original cobalt nucleus. The mirror image (without charge conjugation) is not realized in nature; therefore right-left symmetry (that is, what we call "space-inversion symmetry") is violated. —FIG. 1

successfully applied to the atomic system, which involves only the electromagnetic interaction; later, these symmetry operations were extended to include other phenomena in which not only the electromagnetic interaction but also the strong and the weak interactions participate. It is through these applications in particle physics that the validity of these discrete symmetries was questioned,[2,3] and the questioning led to the discovery of the nonconservation of parity.

The first experiment[4] on parity nonconservation (that is, space-inversion asymmetry) was made on beta decay by Professor C. S. Wu of Columbia, in collaboration with Ernest Ambler, Raymond W. Hayward, Dale D. Hoppes and R. P. Hudson of the National Bureau of Standards (see figure 1). From the same experiment it was deduced that charge-conjugation symmetry is also violated. Immediately afterward the same noninvariance properties were established for pi and mu decay.[5]

In quantum mechanics, the space-inversion operator P is a unitary operator and its eigenvalue is the parity.

If space-inversion symmetry holds, the parity must be conserved. Under P, the state of a particle with momentum k and helicity \( \lambda \) (defined to be its spin component along the direction of k and in units of \( \hbar \) ) transforms to a state of the same particle but with momentum \(-k\) and helicity \(-\lambda\). The operator P satisfies

\[
P|k,\lambda\rangle = \eta_P|-k,-\lambda\rangle
\]

(2)

Where \( \eta_P \) is a phase factor; P also satisfies similar equations for the multiparticle states. Here, the identity of a particle is defined through all of its interactions which include, in particular, its mass, charge and spin.

The suggestion that our known interactions are not strictly invariant under space-inversion symmetry was, at the beginning, based on the theta-tau puzzle[6]

\[
K^+ = \begin{cases} 
\theta^+ \rightarrow \pi^+ + \pi^0 \\
\tau^+ \rightarrow \pi^+ + \pi^0 + \pi^-
\end{cases}
\]

(3)
The two particles \( \theta^+ \) and \( \tau^+ \) were found to be of the same mass and lifetime, suggesting that they are two decay modes of the same particle, the K-meson. However, the parity of the pion had been previously determined through both its strong interaction and its electromagnetic interaction to be \(-1\).

\[
\Psi_{\pi}(\pi) = \Psi_{\gamma}(\pi) = -1
\]

(4)
The subscripts \( \pi \) and \( \gamma \) indicate that the determinations are through \( H_{\pi\pi} \), the strong interaction and \( H_{\pi\gamma} \), the electromagnetic interaction, respectively. Based on the Dalitz analysis, the three pions in the tau decay mode are found to be in a zero-spin state. The same spin value can also be determined through the various production processes for \( \theta^+ \) and \( \tau^+ \). The parity that is determined by strong and electromagnetic interactions must, therefore, be different for the two final states, +1 for the two-pion state and \(-1\) for the three-pion state; consequently we must have parity nonconservation if the \( \theta^+ \) particle is identical with \( \tau^+ \). There now exist numerous experiments that establish that both \( H_{\pi\pi} \) and \( H_{\pi\gamma} \) are invariant under the same space-inversion operation \( P_{\pi\pi} = P_{\pi\gamma} \), but \( H_{\pi\pi} \), the weak interaction, is not. Thus, it is not possible to construct a space-inversion operator P that commutes with the total interaction Hamiltonian H.

At present, the best evidence for \( H_{\pi\pi} \) and \( H_{\pi\gamma} \) being invariant under \( P_{\pi\pi} = P_{\pi\gamma} \) is from experiments in nuclear physics. These experiments[7] establish that for a nuclear level the magnitude of the parity-nonconserving amplitudes is smaller than that of the corresponding parity-conserving amplitudes by a factor of about 10\(^6\).

Indistinguishable particles

If the identity of a particle could be taken for granted it would be possible to define P, the pure
space inversion, and \( T \), the pure time reversal, unambiguously. However, the distinguishability between different particles depends on all of their interactions, and degeneracies often arise if some of the interactions are absent. If the \( P \) and \( T \) symmetries are not valid for all interactions, then their definitions can only be given when some of the interactions are absent; consequently, such definitions are interaction dependent.

If strong and electromagnetic interactions could be switched off, it would not be possible to discover any parity nonconservation from the weak interaction alone. In fact under these hypothetical conditions many otherwise different particles would become degenerate and indistinguishable. As a result of such indistinguishability between particles, one can find a solution for the space-inversion operator under which the presently accepted form of the weak interaction is invariant.

For example, if \( H_{\pi}=H_{\gamma}=0 \), we might infer from reaction 3 that parity is conserved and the parity of the pion is +1. Indeed all known weak interactions are consistent with the assumption that \( H_{\pi}\) is invariant under a different space-inversion symmetry called \( P_{\pi} \). Inasmuch as we regard \( H_{\pi} \) as violating conservation of parity where parity=\( P_{\pi}=P_{\gamma} \), we could also regard \( H_{\pi} \) as parity conserving where parity=\( P_{\pi} \) and attribute the observed nonconservation of parity to violation of \( P_{\pi} \) by strong and electromagnetic interactions.

**Time-reversal invariance**

The question whether our known interactions are or are not invariant under time reversal was raised \(^8\) when the possibility of parity nonconservation was being studied. After the discovery that parity is not conserved, several experiments were performed to test time-reversal invariance in both strong and weak interactions. \(^9,10\) and these experimental results were all consistent with time-reversal invariance. Recently, however, there has appeared an indirect evidence from the \( K_{e}^{0} \) decay that time-reversal symmetry is, like space-inversion symmetry, only approximately valid for the known interactions.

Before discussing this indirect evidence, let us first review the meaning of time-reversal invariance. In quantum mechanics, the time-reversal operator \( T \) is an antunitary operator.

If the theory is invariant under time reversal, then from a solution \( \Psi(t) \) of the Schroedinger equation

\[
\hbar \frac{d\Psi(t)}{dt} = -\frac{1}{2m} H \Psi(t)
\]

we can generate a different solution \( \Psi_{q}(t) \) of the same equation. The \( \Psi_{q} \) is related to \( \Psi \) by

\[
\Psi_{q}(t) = T \Psi(-t) = U_{T} \Psi^{*}(-t)\]

(6)

\( U_{T} \) is a unitary operator in Hilbert space and the asterisk denotes complex conjugation.

An important consequence of time-reversal invariance is the reciprocity relations between the transition probabilities: the magnitude of the \( S \)-matrix element for any transition \( a \rightarrow b \) is equal to that of \( b \rightarrow a \)

\[
|\langle b|S|a\rangle| = |\langle a|S|b\rangle|
\]

(7)

Here \( |a_{q}\rangle = T |a\rangle \) and \( |b_{q}\rangle = T |b\rangle \). Direct tests of such reciprocity relations have been made for several strong reactions, \(^8\) and these tests give good evidence that the strong interaction is invariant under a (certain) time-reversal operation called \( T_{d} \). The upper limit on the ratio of the magnitude of the time-reversal-noninvariant amplitude to that of the time-reversal-invariant amplitude is about 2\% in the proton-triton-deuterium reactions \( p + t \rightarrow d + d \).

As an illustration of how time-reversal invariance can be tested in the weak interaction, we may consider the example of \( \Lambda_{\pi} \) decay

\[
\Lambda_{\pi} \rightarrow N + \pi\]

(8)

The final \( N + \pi \) system can be in either the \( s_{\pi} \) or the \( p_{\pi} \) spin-orbital state with relative amplitudes \( A_s \) and \( A_p \), respectively; the total isospin of the final state is, predominantly, \( I=\frac{1}{2} \). If the weak interaction satisfies the same \( T_{x} \) invariance, then the relative phase \( \phi \), defined by

\[
\frac{A_s}{A_p} = \frac{A_s}{A_p} e^{i\phi}
\]

(9)

is given by

\[
\phi = (\delta_s - \delta_p) \quad (\delta_s - \delta_p) + \pi
\]

(10)

where \( \delta_s \) and \( \delta_p \) are, respectively, the \( s_{\pi} \) and \( p_{\pi} \) phase shifts of the strongly interacting \( N + \pi \) system in the \( I=\frac{1}{2} \) state. The experimental results are \( \delta_s - \delta_p = 7 \text{ deg} \) and \( \delta_s - \delta_p = 15 \pm 20 \text{ deg} \), which are consistent with \( H_{\pi} \) being invariant under the same time-reversal invariance as the strong interaction. The same conclusion is also reached by a similar, but more accurate, experiment on beta decay. \(^10\)

The relative phase \( \theta \) between the Gamow-Teller coupling constant \( g_{\pi} \) and the Fermi constant \( g_{\nu} \) has been measured by M. T. Burgoyne and his collaborators. \(^10\) If the weak interaction satisfies time-reversal symmetry, \( \theta = 0 \) or 180 \text{ deg}. The experimental value is 180 \pm 8 \text{ deg}. [In some theoretical models, however, the weak-interaction strangeness-conserving current \( J_{\mu} \) of the nonleptons is assumed to satisfy]
charge symmetry: $J_\mu^* = -\exp(i\pi/2) J_\mu \exp(-i\pi/2)$

where $J_\mu$ is the $y$ component of the isospin operator.

Under this assumption, independently of time-reversal invariance, $g_A/g_V$ must be real.]

We note that in the $\Lambda^0$ decay, if the initial $\Lambda^0$ is completely polarized, say, along the unit vector $s_A$ in its rest system, then at any given momentum $k$ the final nucleon must also be completely polarized along a direction $s_X$ which is uniquely determined by $s_A$, $k$ and the amplitudes $A_\pi$ and $A_\mu$. Let us now consider the reversed reaction.

$$N + \pi \rightarrow \Lambda^0$$

in which the initial nucleon is polarized along $-s_A$ and its momentum is $-k$. If the system obeyed classical mechanics, time-reversal invariance would imply that the final $\Lambda^0$ in the reversed reaction must be completely polarized along the reversed direction $-s_A$. For the quantum-mechanical system, although the final $\Lambda^0$ in the reversed reaction does remain completely polarized, its direction is, in general, different from $-s_A$ even if time-reversal invariance holds. This elementary property is demonstrated in figure 2.

To produce the final $\Lambda^0$ with its polarization along $-s_A$, in case time-reversal invariance holds, we must not use just the $N_+\pi$ state with the reversed spin and momentum, but we should start with an initially coherent mixture of the appropriate $s_\pi$ and $p_\pi$ incoming wave of $N_+\pi$. Mathematically, such an initial state can be easily obtained by applying the antiunitary operator $T$ onto the final state $\Psi(t=+\infty)$ in the decay $\Lambda^0 \rightarrow N_+\pi$. Physically, however, it seems virtually impossible even to construct the desired coherent time reversed state $T\Psi(t=+\infty)$ for a direct testing of the symmetry (or violation-of-symmetry) of the time-reversal operation. Here lies an important difference between the time-reversal operation in classical mechanics and that in quantum mechanics. In both cases time-reversal invariance means that the time-reversed solutions are always dynamically possible solutions. In classical mechanics such a time-reversed solution becomes an improbable one only for a macroscopic system. In quantum mechanics even a microscopic system is described by an infinite number of variables (that is, by a continuous function of space-time); thus the time-reversed solution for any scattering problem is, in general, an improbable one.

For all practical purposes the only direct and tangible test of time-reversal invariance seems to be the reciprocity relations between various differential cross sections; that is, equation 7 restricted to those states $|a\rangle$ and $|b\rangle$ that represent asymptotically the appropriate initial and final states in which every particle has a definite, but arbitrary, set of
spin and momentum. In this connection it is relevant to mention that all the presently known tests of time-reversal symmetry, such as equation 10, can be derived directly by using the reciprocity relations between the appropriate differential cross sections. The violation of time-reversal invariance means, simply, that such reciprocity relations are not valid.

CPT invariance and particle-antiparticle relations

To understand the recent indirect evidence of time-reversal noninvariance, it is necessary to review the CPT theorem and the experimental evidence for its validity.

In the framework of a local field theory, it can be shown that if a theory is invariant under the continuous group of Lorentz transformations without any discrete element (such as space inversion and time reversal), the theory is automatically invariant under a discrete symmetry operation, called "CPT." If CPT invariance holds for the total Hamiltonian $H$, the matrix element of $H$ between any two states $|A\rangle$ and $|B\rangle$ is related to that between their CPT-conjugate states $|\bar{A}\rangle$ and $|\bar{B}\rangle$ by

$$\langle B | H | A \rangle = \langle \bar{B} | H | \bar{A} \rangle^*$$  \hspace{1cm} (12)

where

$$|\bar{A}\rangle = \text{CPT} | A \rangle$$  \hspace{1cm} (13)

$$|\bar{B}\rangle = \text{CPT} | B \rangle$$

From CPT invariance it follows that if $A$ is a stable particle, then its CPT conjugate $\bar{A}$ is also a stable particle with exactly the same mass. This can be easily proved by setting $A=B$ in equation 12.

The CPT operator is an antiunitary operator; furthermore it relates the state $A$ with a momentum $k$ and a helicity $\lambda$ to that of $\bar{A}$ with the same momentum $k$ but the opposite helicity $-\lambda$. It can be easily shown that the electromagnetic fields $E$ and $H$ are invariant under CPT. Thus by comparing the energy spectrum in an external electromagnetic field, one can prove that $A$ and $\bar{A}$ have opposite charges but otherwise the same electromagnetic form factors (see figure 3). Within the same framework of local field theory it can also be proved that the baryon numbers, or the lepton numbers, of the states $A$ and $\bar{A}$ must be equal in magnitude but opposite in sign. (We assume implicitly that charge, baryon number and lepton number are all strictly conserved.) In the following the state $\bar{A}$ is called the "antiparticle state" of $A$.

Among the mass equalities between particles and antiparticles, the most accurate one is that between $K^0$ and $\bar{K}^0$

$$(K^0 | H | K^0) = (\bar{K}^0 | H | \bar{K}^0)$$  \hspace{1cm} (14)

From the experimental mass difference$^{11}$ $\Delta m$, between $K^0_L$ and $K^0_S$, it is found that equation 14 holds to the accuracy $|\Delta m / m_K| = 10^{-14}$. Thus, we should regard $H_{LL}$, $H_S$, and the strangeness conserving nonleptonic part of $H_{kk}$ to be invariant under CPT.

For the other parts of $H_{kk}$, evidences for CPT invariance come from the lifetime equalities between the states $A$ and $\bar{A}$ which decay through the weak interaction. Such lifetime equalities hold, at least, to the lowest order in $H_{kk}$ and can be easily proved by substituting $H_{kk}$ for $H$ in equation 12.
The present upper limits on such lifetime differences $\Delta \tau$ are given for muons,\textsuperscript{14} pions\textsuperscript{15} and K mesons\textsuperscript{16} by

$$\left| \frac{\Delta \tau}{\tau} \right| < \begin{cases} 0.001 \text{ for } \mu^- \\ 0.08 \text{ for } \pi^+ \\ 0.15 \text{ for } K^+ \end{cases}$$

(15)

Among these, the limit for the strangeness-nonconserving part of the weak interaction is rather inaccurate.

In the following we shall assume that CPT invariance holds; therefore for each particle state $A$ there exists an antiparticle state $\bar{A}$ that has the same physical mass, the opposite charge, the opposite baryon number (or the opposite lepton number) and, if $A$ is unstable, the same lifetime.

**The $C_{st}$ symmetry**

As I remarked earlier, the strong interaction is found experimentally to be invariant under a space inversion $P_{st}$ (to an accuracy of about $10^{-4}$), a time reversal $T_{st}$ (to an accuracy of a few percent) and the CPT operation (to an accuracy of about $10^{-14}$). We can define an operator $C_{st}$

$$C_{st} = (\text{CPT})T_{st}P_{st}$$

(16)

The strong interaction is expected to satisfy the $C_{st}$ invariance to the same accuracy as the $T_{st}$ invariance. Under $C_{st}$ we must have at least approximately, $p \rightarrow \bar{p}$ and $n \rightarrow \bar{n}$, but with their momenta and helicities unchanged; consequently we also have $\pi^+ \rightarrow \pi^-$, $\pi^0 \rightarrow \pi^0$, etc., since the transformation properties of these mesons are determined by those of baryons and antibaryons.

The $C_{st}$ symmetry has also been directly tested\textsuperscript{17} by studying the equality between the energy distributions of $\pi^+$ and $\pi^-$ in the annihilation of protons by antiprotons

$$\bar{p} + p \rightarrow \pi^+ + \pi^- + \cdots$$

(17)

The result puts an upper limit on the $C_{st}$-noninvariant amplitude; it cannot be more than about 1% of the $C_{st}$-invariant amplitude. A similar upper limit of about 2% is obtained by studying the energy distributions of $K^+$ and $K^-$ in the same (proton-antiproton) annihilation experiment.

However, the weak interaction is known to violate $C_{st}$ invariance. This can be inferred by using the properties that $H_{st}$ has large violations of $P_{st}$, but it is (at least to a good approximation) invariant under $C_{st}P_{st}T_{st} = CPT$ and $T_{st}$. The same conclusion can also be directly reached by considering $K^0$ decays. Let $K^0$ be the neutral K meson with the short lifetime and $K^0$ that with the long lifetime. If $C_{st}$ symmetry were conserved in their weak decay, $K^0_t$ and $K^0$ would be eigenstates of $C_{st}$, and their $C_{st}$ eigenvalues must be of opposite signs. But the decays $K^0_t \rightarrow 2\pi$ and $K^0 \rightarrow 3\pi$ are both in the $C_{st} = +1$ states. Thus $C_{st}$ invariance must be violated, and the $C_{st}$ symmetry is not exact. If $|p|$ is the physical proton state, the state $\text{CPT}|p|$ is the physical antiproton state, but the state $C_{st}|p|$ is only approximately the antiproton state.

**Two-pion decay of neutral K mesons**

It was discovered last year by Christenson, Cronin, Fitch and Turlay\textsuperscript{18} that the long-lived component $K^0_S$ of the neutral K meson has a two-pion decay mode

$$K^0_S \rightarrow \pi^+ + \pi^-$$

(18)

Since both $C_{st}$ and $P_{st}$ (in the center-of-mass system) interchange $\pi^+$ and $\pi^-$, the final (two-pion) system in this decay must be of $C_{st}P_{st} = +1$. On the other hand, the same long-lived component $K^0_L$ also decays into three pions

$$K^0_L \rightarrow \pi^+ + \pi^- + \pi^0$$

(19)

All pions produced are observed to be predominantly in the $s$ state. Thus, the final three-pion state is (or at least predominantly is) of $C_{st}P_{st} = -1$.

Now, $K^0_S$ is, by definition, a particle with a definite lifetime. That it can decay into different states with opposite $C_{st}P_{st}$ values shows conclusively that $C_{st}P_{st}$ is not conserved in the $K^0_S$ decays. The $C_{st}P_{st}$ violation is, therefore, established independently of the detailed theory of the $K^0_S$, $K^0_L$ system.

The observed $C_{st}P_{st}$ noninvariance implies that

$$[H, C_{st}P_{st}] \neq 0$$

(20)

where $H$ represents the total interaction. In this decay, both the initial and the final states are eigenstates of $H_{st}$ and the transition is due to $H_{st}$. This means that the $C_{st}P_{st}$ violation could be due to the strong interaction\textsuperscript{19} or to the electromagnetic interaction\textsuperscript{20} or to the weak interaction,\textsuperscript{21} or a combination of these interactions, or the presence of some new interactions, such as the superweak interaction,\textsuperscript{22} which, however, I will not discuss further here. From this single experiment, it is not possible to decide which interaction is responsible for this violation.

If we assume the validity of CPT invariance in the $K^0_S$ decay, then it follows from equations 16 and 20 that $T_{st}$ invariance is also violated; that is

$$T_{st}HT_{st} \neq H$$

(21)

The amount of the observed $C_{st}P_{st}$ violation in
the $K^0_2$ decay is a small one, characterized by a parameter

$$\epsilon = \frac{\text{Rate} (K^0_2 \to \pi^0 + \pi^-)}{\text{Rate} (K^0_2 \to \pi^+ + \pi^-)} \approx 2 \times 10^{-5} \quad (22)$$

$K^0_2$ is the short-lived component of the neutral $K$ meson. The smallness of this parameter is, perhaps, one of the most puzzling features of this new discovery. To explain $\epsilon$, we may invoke a small $C_{\text{st}}$-nonvariant term in $H_{\text{st}}$, or, if $\epsilon$ is a typical example of a $C_{\text{st}}P_{\text{st}}$-nonvariant amplitude, a small $C_{\text{st}}P_{\text{st}}$-nonvariant term in $H_{\text{wk}}$. There is no difficulty in doing either of these; one is only puzzled by the smallness of such violation and by the multitudes of different ways to construct such a violation.

An alternative, and perhaps more attractive, possibility is to assume that $H_{\text{st}}$ and $H_{\text{wk}}$ are invariant under $C_{\text{st}},$ but $H_{\gamma}$ has a large violation of $C_{\text{st}}$ invariance. In this case all strong and weak processes can have a small $C_{\text{st}}$- and $C_{\text{st}}P_{\text{st}}$-nonvariant amplitude through virtual emission and absorption of photons. In particular $K^0_2$ can decay into two pions with a fractional amplitude $\epsilon = \alpha/\pi$, (where $\alpha$ is the fine-structure constant) and this gives a natural explanation of the smallness of the observed $C_{\text{st}}P_{\text{st}}$-nonvariant amplitude.

This last possibility naturally raises many questions; among these the most urgent one is whether the hypothesis of $H_{\gamma}$ being nonvariant under $C_{\text{st}}$ is already in contradiction with some other known experiments. In an extensive study\textsuperscript{28} that I have made in collaboration with Jeremy Bernstein and Gerald Feinberg, we find that there exists at present no experimental evidence that $H_{\gamma}$ of the nonleptons is, or is not, invariant under $C_{\text{st}},$ nor is there any evidence that $H_{\gamma}$ satisfies, or does not satisfy, the same time-reversal invariance $T_{\gamma}$ as the strong interaction. If the electromagnetic interaction is noninvariant under $C_{\text{st}},$ since $H_{\gamma}$ is experimentally found to be invariant under both $P_{\text{st}}$ and CPT, it follows that $H_{\gamma}$ must also violate $T_{\text{st}}$ invariance.

The possibility that $H_{\gamma}$ may violate $C_{\text{st}}$ and $T_{\text{st}}$ invariances is, of course, only a theoretical possibility. Nevertheless, the present absence of evidence that $H_{\gamma}$ is, or is not, invariant under $C_{\text{st}}$ and $T_{\text{st}}$ should provide sufficient incentive for further experimental efforts in this direction. Various tests have been proposed, and some of these are already in progress.

**What is charge conjugation?**

The electromagnetic interaction of the leptons is well known to be invariant under charge conjugation $C_{\gamma}$. Furthermore we know that the minimal electromagnetic interaction of any system of spin-zero and spin-one-half particles is always invariant under charge conjugation; it is also invariant under a time reversal $T_{\gamma}$ and a space inversion $P_{\gamma}$. The electromagnetic interaction of the leptons is well described by such a minimum interaction. It seems, therefore, aesthetically appealing to assume that there exists a charge conjugation operation $C_{\gamma}$ under which all electromagnetic currents change sign and that all electromagnetic interactions, including that of the nonleptons, are invariant under this charge conjugation symmetry $C_{\gamma}$. From the experimental evidences of $P_{\text{st}}$ and CPT invariances, we know that $H_{\gamma}$ must also be invariant under $P_{\gamma}$ and $T_{\gamma}$ where

$$P_{\gamma} = P_{\text{st}}$$

$$C_{\gamma}P_{\gamma}T_{\gamma} = C_{\text{st}}P_{\text{st}}T_{\text{st}} = \text{CPT}$$

The question whether the electromagnetic interaction does, or does not, satisfy the $C_{\text{st}}$ symmetry can be simply viewed as whether the charge-conjugation operator $C_{\gamma}$ is, or is not, the same as the operator $C_{\text{st}}$. (The $C_{\text{st}}$ may, for example, be regarded as the baryon-number conjugation, and can, in principle, be different from the charge conjugation.) If the electromagnetic interaction does violate the $C_{\text{st}}$ symmetry, then $C_{\text{st}} \neq C_{\gamma}$ and, therefore $T_{\text{st}} \neq T_{\gamma}$.

As I have already remarked, the presently known form of weak interaction $H_{\text{wk}}$ is invariant under its own space inversion $P_{\text{wk}}$; it can also be shown that $H_{\text{wk}}$ is invariant under a time reversal $T_{\text{wk}}$ and a $C_{\text{wk}}$ conjugation.\textsuperscript{7} Indeed, all our experimental results are consistent with the assumption that each of these interactions $H_i$ is invariant under its own $C_i$, $P_i$ and $T_i$, where $i = \text{st}$, $\gamma$ and wk, and

$$C_i P_i T_i = \text{CPT} \quad (23)$$

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A possible mismatch pattern for $C_{\gamma}$, $P_{\gamma}$ and $T_{\gamma}$ where $i$ stands for wk, $\gamma$ or st

<table>
<thead>
<tr>
<th>$H_{\text{st}}$</th>
<th>$C_{\text{st}}$</th>
<th>$T_{\text{st}}$</th>
<th>$P_{\text{st}}$</th>
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</thead>
<tbody>
<tr>
<td>$H_{\gamma}$</td>
<td>$C_{\gamma}$</td>
<td>$T_{\gamma}$</td>
<td>$P_{\gamma}$</td>
</tr>
<tr>
<td>$H_{\text{wk}}$</td>
<td>$C_{\text{wk}}$</td>
<td>$T_{\text{wk}}$</td>
<td>$P_{\text{wk}}$</td>
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</tbody>
</table>

$C_{\text{st}}P_{\text{st}}T_{\text{st}} = C_{\gamma}P_{\gamma}T_{\gamma} = C_{\text{wk}}P_{\text{wk}}T_{\text{wk}} = \text{CPT}$
The well known parity nonconservation is due to the mismatches
\[ P_{\nu\tau} = P_{\gamma} = P_{\text{st}} \quad C_{\nu\tau} = C_{\gamma} \quad C_{\nu\tau} = C_{\text{st}} \]

It remains an open question whether the recent discovery of the \( C_{\nu}\) nonconservation in \( K^0 \to \pi^+ + \pi^- \) can also be attributed to a similar mismatch—one between \( C_{\nu}\) and \( C_{\gamma}\). A possible pattern of such mismatches is given in the table on page 30.

Our concept of "C" started with the charge conjugation \( C_{\nu}\) determined by the electromagnetic interaction of the electron.23 Later, the operator \( C_{\gamma}\) was extended to other interactions and was called "particle-antiparticle conjugation." After the discovery of parity nonconservation, it was already known that charge-conjugation invariance cannot be extended to the weak interaction and that the concept of particle and antiparticle rests, instead, on CPT invariance. Nevertheless, the hypothesis that charge-conjugation invariance \( C_{\nu}\) is applicable to the strong interaction was assumed without question, and that the electromagnetic interaction and the strong interaction satisfy the same time-reversal invariance was taken for granted.

The progress of science has always been the result of a close interplay between our concepts of the universe and our observations of nature. The former can only evolve out of the latter, and yet the latter are also conditioned to a remarkable degree by the former. As we expand our fields of observation, naturally, we also extend our basic concepts. At times, these two factors, the concept and the observation, may become so interlocked that even some of the fundamental principles used in an entire domain of familiar phenomena may, to our chagrin, turn out to have no actual experimental basis. The history of these discrete symmetries has been a particularly rich one, full of such surprises.

References